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COMMUNICATION THROUGH RANDOM MULTIPATH MEDIA

by
D.P.HARRIS

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Lockheed

MISSILES & SPACE COMPANY

A GROUP DIVISION OF LOCKHEED AIRCRAFT CORPORATION

SUNNYVALE, CALIFORNIA

NOTICE

FOREWORD

The work described in this report was initially supported by a National Science Foundation Graduate Fellowship. Subsequent studies and the writing of the report were supported by the Independent Research Program at Lockheed Missiles & Space Company. Results were submitted in a dissertation to the department of Electrical Engineering, Stanford University, and were published as Technical Report No. SEL-62-031 by Stanford Electronics Laboratories, Stanford University, under Office of Naval Research Contract Nonr 225 (24), NR 373 360.

ABSTRACT

The performance capabilities of some specific techniques for communicating through noisy, randomly time-varying multipath channels are considered. Analysis is undertaken in sufficient depth to give guidance in the selection of design parameters of various types of systems. The results permit some comparisons to be made of the performance potentials of different communication techniques.

Bounds on the communication rate possible with an adaptive matched-filter receiving technique are obtained as functions of the channel-sounding effort, the signal characteristics, and the channel characteristics. These bounds are found to be relatively insensitive to the degree of effort expended in measuring the channel transfer function.

The performance characteristics of a class of radiometric signal-detection techniques that require no channel-sounding provisions are analyzed. Detailed design equations and graphs for determining and optimizing performance of such techniques are presented. The results appear to be very attractive when relatively large channel bandwidths are available.

An investigation of the performance of some elementary narrowband multipath communication techniques indicates that they may offer competitive performance potentials under some conditions. For many types of multipath channels, the advantages of seeking generally optimum communication techniques are somewhat limited. With improved design of suboptimum techniques, it is possible to achieve results reasonably close to ultimate theoretical performance limitations.

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SYMBOLS AND CONVENTIONS

A partial list of the symbols used in this report and their definitions appear below. Additional definitions appear in the text and illustrations.

e	natural base of logarithms
$e(t)$	a time-varying error function
k	fraction of total transmitter power used for sending independent channel-sounding signals; general index of summation
m	size of signalling alphabet (i.e., number of symbols)
n	number of taps on a delay-line channel model; additive noise signal; number of degrees of freedom of a chi-square probability distribution
$n(t)$	additive noise signal
n_o	power-spectral density of additive receiver noise, in watts per cps
$\hat{x}(t)$	total transmitted signal
$x(t)$	message-bearing component of transmitted signal
$x_s(t)$	channel-sounding component of transmitted signal
$x^{(j)}(t)$	the j^{th} member of a set of m possible signals that may be transmitted in time interval T
B_s	doppler-spread bandwidth of received signal when a single-frequency sine wave is transmitted over a random multipath-propagation circuit
F_P	power-requirement factor of a communication system, expressed in terms of received-signal energy per bit of information communicated, i.e., $F_P = S/n_oR$
F_{P_o}	optimum (minimum) power-requirement factor for a given type of system and channel
$H(y)$	entropy per second, or per degree of freedom, of a stochastic function y, as appropriate [see Eq. (2.2)]
$H_x(y)$	conditional entropy of y, given x [see Eq. (2.2)]

SYMBOLS AND CONVENTIONS (Cont'd)

N	additive-noise power affecting a system, or subsystem
$P(x)$	probability distribution of a stochastic function x (normally a density; discrete probabilities are included through use of impulse functions)
R	rate of communication of message information, in bits per second
S	average received-signal power during a communication period
$\hat{S}$	average received-signal power during a pulse-observation period
$S(x)$	average power of a function $x(t)$ (normalized to equal mean-square amplitude, and hence, equal to the variance of a zero-mean signal)
$\overline{S}(x)$	entropy power of a function $x(t)$ [see Eq. (3.3)]
$\overline{S}(x)\|_y$	conditional entropy power of $x(t)$, given $y(t)$
$S(x)\|_y$	conditional variance, or power, of $x(t)$, given $y(t)$
SNR	signal-to-noise power ratio, generally equal to S/n_oW for a received signal
$\widehat{SNR}$	signal-to-noise ratio of signal during a pulse-observation interval
T	duration of a transmitted symbol, or pulse, in seconds
T_m	time dispersion of a channel, i.e., maximum differential multipath-propagation delay
T_R	average repetition time of symbol, or pulse
T_S	propagation delay corresponding to shortest propagation path
W	total bandwidth occupied by a system (or subsystem), in cps
W'	total bandwidth occupied by message-signal components of a signal
W/R	bandwidth-requirement factor of a communication system, in cps per bit per second
$\vec{Y}(f)$	complex frequency response of a linear filter
γ	mean SNR of delay-line tap measurement [see Eq. (3.9)]; dummy variable of integration

SYMBOLS AND CONVENTIONS (Cont'd)

σ_r^2	variance of a function $r(t)$
τ_d	tap spacing of a tapped delay line, in seconds
ϵ	probability of receiver decision error per symbol, i.e., error rate
$\mathcal{F}_P$	power-requirement factor expressed in terms of transmitted bit rate, i.e., $\mathcal{F}_P = S/n_o\mathfrak{R}$
$\mathcal{F}_{P_o}$	minimum value of $\mathcal{F}_P$, for specified conditions
Γ	gamma function
$\mathfrak{R}$	maximum rate at which message information may be encoded for transmission with a given modulation format, irrespective of loss due to errors in the receiving process

I. INTRODUCTION

There are presently available many multipath- and scatter-communication circuits that may be characterized in the following manner:

1. When a signal waveform is transmitted over the circuit, a delayed, attenuated, and distorted version thereof appears at the receiving terminals.
2. The distortion of the signal may be represented by the operation of a finite-memory, linear, randomly time-varying filter plus the addition of noise from an independent source.

The technical problems involved in conveying intelligence over such circuits become more difficult with increasing filter variation rates and with increasing differential filter-memory duration. Since the early days of radio communication it has been possible to avoid poorly behaved propagation circuits in many cases by carefully predicting and selecting the frequency of operation. The rapidly growing needs for communication facilities, however, are demanding that better use be made of all available circuits, including those with less desirable properties.

Most of the recent advances in practical communication systems for use on poorly behaved circuits have been based on refinement and automation of older techniques--such developments as better detectors, better diversity-combining techniques for fading channels, error-correction coding techniques, automated propagation sounders and channel selectors, and automation of information-feedback processes. In the meantime, a sizable effort has been devoted to a logical design procedure involving system synthesis from the very foundations, rather than elaborations on previously established techniques. Heretofore, system-synthesis efforts have been concerned primarily with the solution to the forward multipath circuit alone, without the added complication of information-feedback processes.

The application of modern methods of statistical analysis to theoretical multipath system-synthesis problems has been appropriate because of the inherently stochastic nature of typical channel perturbations. While the use of statistical methods has led to a better understanding of multipath-communication problems and to specific conceptual system designs, the successful reduction of these results to practice has been slow and erratic. Further progress in this direction is apparently impeded by the following difficulties:

1. Most theoretical investigations of random-multipath communications have not treated the problems of performance evaluation in sufficient detail to justify and guide the subsequent development of practical systems. While it is very desirable to derive and propose functional forms of systems that are optimum or desirable in one sense or another, there is also a distinct need to follow such proposals with analyses indicating how much performance improvement can be expected, under specific operating conditions, relative to techniques already in use. The development engineer requires a reasonable promise of worthwhile results in order to justify his efforts, and he also needs guiding criteria for selecting various design parameters. The complex nature of such systems generally renders the optimization of uncertain parameters by purely experimental methods a prohibitively laborious procedure.
2. In most cases, multipath system-synthesis procedures that have been developed involve an assumption that certain a priori correct knowledge of short-term channel behavior is available at the receiver. In practice this knowledge is never precisely correct; the effects of channel-measurement errors need to be considered, as well as the influence of channel-sounding requirements on the overall system-optimization problem. The unsolved problems involving channel measurements were recognized clearly in the work of Price [Ref. 1] and Turin [Ref. 2]. In the more recent work of Kailath [Ref. 3] some similar difficulties arise in connection with an assumption that a time-varying correlation matrix pertaining to the channel is known a priori at the receiver.
3. The problems of performance analysis and channel-sounding uncertainties need to be treated in connection with channel models that realistically represent measurable characteristics of troublesome multipath-propagation phenomena.

There has been considerable need for dealing directly with the above problems in evaluating the usefulness of newly discovered scatter-propagation phenomena, as well as in making better use of the more familiar ionospheric-multipath circuits. The results presented in the following sections have been pursued with a primary goal of bridging some of these remaining gaps between statistical communication theory and its successful application to practical multipath- and scatter-communication problems.

In Chapter II the random-multipath communication problem and its optimum solution are formulated in general terms, and consideration is given to some of the mathematical difficulties involved in reducing the solution to practice and determining the resulting performance. Attention is directed to the performance potentials of a class of techniques involving adaptive matched-filter reception in Chapter III. Detailed consideration is given to the overall system-optimization problem and to the limitations imposed by errors in a channel-sounding process. A number of performance curves are presented for these techniques under various conditions of propagation.

In Chapter IV consideration is given to the performance of a class of multipath techniques that do not require measurements of the time-varying transfer characteristic of the propagation circuit. The use of such techniques is apparently necessary whenever the channel behavior is so poor that satisfactory channel measurements cannot be made. Detailed analyses of these techniques are carried out, and extensive design data and performance curves are presented.

In order to determine how much improvement the techniques of Chapters III and IV can offer over simpler techniques based on extensions of present practice, a study of some of the latter is included in Chapter V. An ultimate bound on the performance of future techniques is determined, and a number of representative performance curves are illustrated and compared for the various techniques that have been considered.

II. SOME PRINCIPLES OF RANDOM-MULTIPATH COMMUNICATION

The mathematical theory of communication, as developed by Shannon and others, is generally applicable to all types of practical communication channels. In the study of random-multipath-communication problems it is desirable to apply communication theory to the following system model:

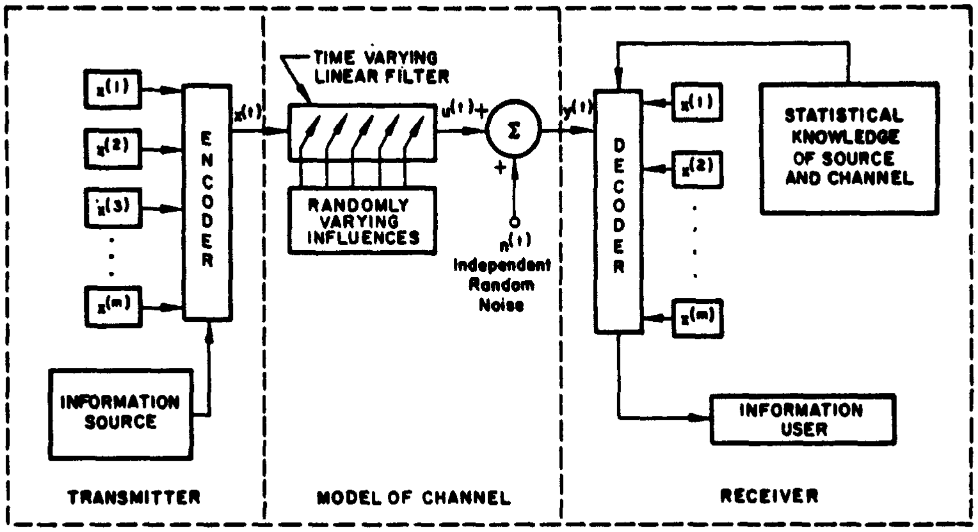


FIG. 1. BLOCK DIAGRAM OF A RANDOM-MULTIPATH COMMUNICATION SYSTEM.

A communication system designed to operate in conjunction with the indicated channel model can be expected to have certain a priori long-term statistical knowledge of the random processes that affect the relation between $x(t)$ and $y(t)$. If there is to be any information of a more precise, short-term nature concerning the particular processes that operate on $x(t)$ to produce $y(t)$ in some particular interval of time, such information must generally be obtained directly from observations made on $y(t)$ itself. If short-term channel knowledge is to be used in the message-encoding process at the transmitter, then a return path or feedback circuit of some sort must be provided, and the associated delays and uncertainties must be considered. It generally will be assumed herein that the encoding process is to be based solely on available long-term statistical knowledge of the channel. A brief discussion of some uses of feedback is included in Chapter VI.

A. APPLICATION OF COMMUNICATION THEORY TO MULTIPATH CHANNELS

A communication system may be defined as a combination of transmitter, channel, and receiver having the general form indicated in Fig. 1. The output of the information source may take the form of a sequence of symbols drawn independently from a finite alphabet. The encoder translates these symbols into a new sequence of symbols, of individual duration T, drawn from an allowed set of symbol waveforms $\{x^{(j)}(t)\}$, $j = 1, 2, 3, \ldots m$, which are appropriate for the channel being used. The channel typically distorts the transmitted symbols such that observation of the received symbols is not sufficient to determine with certainty which symbols were transmitted. In order to determine the sequence of symbols originating at the information source, the receiver must guess, in some suitable manner, what sequence of transmitted symbols might have produced the observed results, and translate the sequence of guesses back into the symbol language of the information source.

The phenomena producing the output of the information source must involve *a priori* uncertainty from the receiver's point of view, if the channel is to convey information. Since such symbol sequences, as well as the signal-perturbing qualities of the random channel, are random processes, it is appropriate for the criteria of system performance to involve probabilities. Given a particular perturbed y sequence, the allowed alphabet $\{x^{(j)}(t)\}$, and *a priori*-available statistical knowledge of channel behavior, it appears that the best receiver should be the one which minimizes the probabilities of error in the sequence of guesses concerning the $x^{(j)}$.

In the mathematical theory of communication [Refs. 1, 2], the randomness, or uncertainty, of the output x from a stochastic source is described in terms of its average entropy $H(x)$ per unit time (or per degree of freedom where the total number of degrees of freedom is directly proportional to time duration of x). Equivocation, or conditional entropy $H_y(x)$, is indicative of the uncertainty in x when perfect a priori knowledge of a related signal y is available. The probability of error in the sequence of guesses made by the receiver of Fig. 1 is clearly related to this conditional uncertainty.

In general, as symbol duration T is increased with alphabet size m and various system power levels held fixed, the probability of error in receiver decisions can be made arbitrarily small, but the amount of information encoded into the transmitted signal per unit time likewise diminishes. Shannon has shown, however, that for a suitable (or allowable) information-source rate it is possible to increase m exponentially with T such that the rate of transmission of information remains constant, and yet probability of error still diminishes to an arbitrarily small value. If the transmitter alphabet is generated by drawing symbols at random from a stochastic waveform generator, the allowable source rate has a maximum limiting value called system capacity C, which may be found by computing entropies when the given waveform generator drives the channel directly. The maximum value of C, computed for the best possible choice of waveform generators within specified constraints on power and bandwidth, is called channel capacity C_c. C_c is the upper bound on the allowable source rate at which information can be transmitted over the channel by any coding scheme, and is indicative of the ultimate performance possible, with constraints, when complete freedom to optimize the transmitter and receiver is assumed.

It is mathematically expedient to consider transmitted and received signals that are essentially constrained to occupy no more than a specified band of frequencies W. As shown by Shannon [Ref. 4], they can then be specified, for a time T, by approximately $2TW$ numbers, and their statistical structure may be adequately specified by finite-dimensional probability-distribution functions. The statistics of the received signal, for example, are thus determined by

$$P(y_1, y_2, y_3, y_4, \dots y_n) \triangleq P(y). \qquad (2.1)$$

Shannon [Ref. 4] has derived the following expressions for the rate of transmission R of information through any channel whose input x and output y may be characterized by the joint, finite, probability-distribution function P(x, y):

$$R = H(x) - H_y(x) = H(y) - H_x(y) \tag{2.2}$$

where

$$H(x) \triangleq -\frac{1}{T}\int_x P(x) \log P(x)dx$$

and

$$H_y(x) \triangleq -\frac{1}{T}\int_x\int_y P(y)P(x/y) \log P(x/y)dxdy$$

This can be written in the form

$$R = \frac{1}{T}\iint P(x,y) \log \frac{P(x,y)}{P(x)P(y)} dxdy \tag{2.3}$$

The channel capacity C_c is defined as the maximum rate that can be achieved for the very best possible distribution of transmitted signals, P(x), as symbol time duration T grows without limit. That is,

$$C_c \triangleq \lim_{T\to\infty}\left\{\max_{P(x)}\left[\frac{1}{T}\iint P(x,y) \log \frac{P(x,y)}{P(x)P(y)} dxdy\right]\right\} \tag{2.4}$$

The channel itself is completely specified by the conditional probability distribution P(y/x). Thus, given a probability distribution P(x) of the possible transmitted signals,

$$P(y) = \int_x P(x,y)dx = \int_x P(x)P(y/x)dx \tag{2.5}$$

Furthermore, since P(x,y) = P(x)P(y/x) = P(y)P(x/y), P(x/y) is also completely determined by specifying P(x) and P(y/x).

In the application of the theory it may not always be desirable to deal directly with individual signal waveforms as has been done here. With the techniques of Chapter IV, for example, it is more convenient to deal collectively with certain groups of waveforms and to characterize all the members of each group by a single number depicting energy.

The solution of multipath-communication problems by application of the above theory may be broken down into the following tasks, given available long-term statistical information pertaining to the channel:

1. Finding a set of signals $\{x^{(j)}(t)\}$, $j = 1, 2, \dots m$, which, within specified constraints on number, duration, and energy, will yield a desired rate of communication R.
2. Devising a logical procedure for selection of j in accordance with the content of the message to be communicated (i.e., designing an encoder, and consequently a decoder).
3. Devising a logical procedure for making the best receiver "guess" as to which one of the $x^{(j)}(t)$ was transmitted in an interval of time T, based on observation of $y(t)$ in an appropriate corresponding time interval.

Useful developments from past multipath-system research efforts have fallen almost exclusively into the realm of tasks 2 and 3. Turin [Ref. 2] has obtained some results pertaining to task 1, but only for a degenerate multipath model possessing a single path and under the normally unrealizable condition that short-term channel information be available from sources other than $y(t)$ itself. In many respects, however, task 1 is the most vital of the three. Even if one assumes that tasks 2 and 3 are to be accomplished in a wholly optimum manner, the resulting system performance is completely indeterminate until the transmitted waveforms have been specified.

Let us consider some of the reasons why it may be difficult to apply communication theory to these tasks in the case of practical multipath-channel models. With regard to Fig. 1, it is found that

$$P(y/x) = \int_u P(u/x)P(y/u)du \tag{2.6}$$

For a large, important class of channels the additive noise is, or can be satisfactorily represented by, a white gaussian random variable of bandwidth W. In such a case,

$$P(y/u) = P(n) = \prod_{i=1}^{2TW} \left\{ \frac{1}{\sqrt{2\pi\sigma_n^2}} \exp\left[- \frac{(y_i - u_i)^2}{2\sigma_n^2} \right] \right\} \tag{2.7}$$

If the transfer function of the linear filter in the channel model is known precisely a priori, then $P(u/x)$ is for each $x^{(j)}(t)$ a unit impulse at some known signal $u(t)$. The integration of Eq. (2.6) is thus trivial and $P(y/x)$ is obtained immediately from Eq. (2.7). Furthermore, Shannon [Ref. 5] has shown that such a conditional-probability distribution is a function only of the total energy in the difference waveform $y(t) - u(t)$, and does not otherwise depend on the individual values of the numbers describing these two signals. In selecting a set of transmitted signals $\{x^{(j)}(t)\}$ to maximize $R = H(y) - H_x(y)$, it is found that $H_x(y)$ is independent of $P(x)$. To maximize R, it is only necessary to seek transmitted-signal waveforms that maximize $H(y)$.

There is still much to be learned about optimum selection of transmitted signals for a channel with a priori-known transfer function. However, once either a systematic or an arbitrary choice of transmitted symbols has been made, the necessary probability distributions are likely to be manageable; and the problems of calculating R, designing transmitter encoding logic and receiver decision processes, and calculating receiver decision-error rates may be handled in a systematic manner. Furthermore, the physical equipment for instrumenting and analyzing such a system need not be unduly elaborate.

But there is a drastic change in the situation if the transfer function of the filter in the channel model cannot be known a priori, as is often the case for practical multipath circuits. Now both $H(y)$ and $H_x(y)$ may vary in complex manners as the composition of the allowable set of transmitted signals is varied, and no systematic procedures for maximizing their difference have yet been developed. Furthermore, the conditional probability distribution $P(u/x)$, based on available long-term statistical channel knowledge, has become for each and every $x^{(j)}(t)$ a distinct and exceedingly complex scalar field in a space of $2TW$ dimensions, with $2TW$ likely to be a very large number. Evaluation of this field from the available statistical channel information generally requires a laborious individual calculation for each pertinent point u. The evaluation of $P(y/x)$ from Eq. (2.6), even just for a single pair of points x and y in their corresponding multidimensional spaces, becomes a formidable computational task. When one considers that this latter probability distribution needs to be adequately specified, investigated, and manipulated everywhere in a space of 4TW dimensions, it is not surprising that analytical progress has been limited.

With regard to large classes of available multipath propagation circuits, it is presently difficult to envision mathematical advances that will permit channel capacity to be determined and systems to be generally optimized and instrumented. Nevertheless, in the technical literature there are many ideas, proposals, and developments for combating multipath-communication problems. Some of these systems are soundly based on logical extensions of techniques that are indeed optimum for interesting extremes of channel behavior. But a common limitation pertaining to most of these techniques has been an inadequacy of analytical and experimental knowledge to be used in evaluating, comparing, and designing them for operation over specific types of available multipath-communication circuits.

Several of the more promising classes of multipath-communication techniques have been selected for detailed performance study in the following chapters of this report. The first of these, the adaptive matched-filter receiver, can be readily synthesized as an optimum configuration where the precise transfer function of the linear-filter portion of the channel model can be known a priori without a need for continuous channel sounding. Answers are sought to the following questions concerning the application of such a system to a practical multipath-propagation circuit:

1. What criteria are appropriate for describing the performance of such a system?
2. What is the effect of channel-measurement errors on the performance?
3. In what manner do channel-measurement errors depend on system-design parameters?
4. What compromises must be made to optimize the overall performance of the system, and how sensitive is this procedure to the variation of design parameters?
5. What potential advantages does the use of such a system offer relative to other possible multipath-communication techniques?

The second class of techniques to be studied does not involve any explicit channel-measurement processes. From an extension of Price's [Ref. 6] analytical results for a very rapidly fluctuating single-mode scatter circuit, it may be concluded that simple radiometric signal-detection techniques are generally optimum when channel behavior is so poor that reasonably valid transfer-function measurements cannot be made. In investigating the performance of these techniques, answers to questions 1, 4, and 5 above are sought.

Finally, in order to interpret the significance of the results obtained for the above classes of techniques, answers to similar questions are needed concerning the performance potentials of some representative techniques based on present multipath-communication-system practice.

B. CRITERIA FOR SYSTEM PERFORMANCE

In order to make evaluations and comparisons of the desirability of using some particular communication technique, it is first necessary that one or more appropriate performance criteria be selected. Such criteria should be related as closely as possible to the potential communication effectiveness of a system when specified constraints are imposed on its manner of operation. Such constraints might involve the available transmitter power, the bandwidth of the channel available for occupancy, the makeup of transmitted signals, and the type of receiver. Constraints might also be placed on numerous other details of system operation.

<u>Channel capacity</u> is obviously not a suitable system-performance criterion, in the present instance, because it is too difficult to compute and because it is completely independent of the particular characteristics of any and every communication technique. Expressions for channel capacity, or bounds thereon, are useful primarily as indications of the improvement possible in system performance where all constraints are removed except those on power and bandwidth. <u>System capacity</u> is a more suitable performance criterion, since it reflects the limitations of transmitted signals with nonideal characteristics. However, even this criterion implies a freedom to use the ideal

probability-computing receiver, and thus cannot reflect all of the constraints that might be desired. A more appropriate criterion for system evaluation is simply the maximum communication rate R achievable, subject to arbitrary constraints on transmitter and receiver operation, and subject to detailed specification of channel characteristics. This criterion reflects the maximum rate at which information can be fed continuously into the transmitter of a particular system and recovered at the output of the receiver.

The type of constraints imposed on a system will have a direct effect on the way that R is determined. If there is freedom to use a large number of alphabet symbols drawn at random from a stochastic signal source, then a procedure may be used which is similar to that used in calculating system capacity. Provided a correction is made for loss of information in the nonideal receiver, R may be calculated directly from the entropies of transmitted and received signals when the stochastic signal source is substituted for the transmitter.

An alternate procedure may be used for finding R where a constraint is placed on the number of distinct waveforms (or classes of waveforms) which may be used as symbols. If the decision-error probabilities of the receiver are first determined, then the system may be regarded as a noisy, discrete, digital channel. R may then be determined from the communication theory of discrete channels [Ref. 4]. In the case of alphabet size m, symmetrical error probability ϵ, and a priori equal symbol probability $1/m$, for example,

$$R = H(x) - H_y(x)$$

$$= \frac{1}{T}\left\{\sum_{j=1}^{m} P(x_t = x^{(j)}) \log\left[\frac{1}{P(x_t)}\right] - \sum_{i=1}^{m}\sum_{j=1}^{m} P(x_r = x^{(i)}) P(x_t = x^{(j)}/x_r) \log\left[\frac{1}{P(x_t/x_r)}\right]\right\}$$

$$= \frac{1}{T}\left[(m)\left(\frac{1}{m}\right)\log(m) - \underbrace{m\left(\frac{1}{m}\right)(1-\epsilon)\log\left(\frac{1}{1-\epsilon}\right)}_{\substack{\text{Double summation}\\ \text{term for } i=j}} - \underbrace{m(m-1)\left(\frac{1}{m}\right)\left(\frac{\epsilon}{m-1}\right)\log\left(\frac{m-1}{\epsilon}\right)}_{\substack{\text{Double summation}\\ \text{term for } i\neq j}}\right]$$

$$= \frac{1}{T}\log_2(m)\left[1 - \frac{(1-\epsilon)\log\left(\frac{1}{1-\epsilon}\right) + \epsilon\log\left(\frac{m-1}{\epsilon}\right)}{\log(m)}\right] \tag{2.8}$$

where $x_t^{(j)}$ is the transmitted symbol and $x_r^{(i)}$ is the receiver guess.

Since it is frequently the practice to describe digital-system operation in terms of the maximum allowable information-encoding rate irrespective of channel equivocation, which is specified separately by giving ϵ, the symbol $\mathfrak{R} \triangleq \frac{1}{T} \log_2(m)$ will occasionally be used for this purpose. In the case of symmetrical error probabilities, R may be readily determined from Eq. (2.8) given $\mathfrak{R}$, m, and ϵ.

The final results of analyzing various communication techniques for use on a channel of given characteristics are generally expressible in terms of communication rate R as a function of average received signal power S and signal bandwidth W. The number of parameters so involved may be reduced from three to two by expressing S and W in terms of per-unit-R values. It can be seen that the "size" of a communication system may be increased by applying equal integral scaling factors respectively to S, W, and R. Such a result may be realized by simply duplicating the former system the integral number of times on adjacent frequency bands without any basic design modification or any change in the normalized performance criteria. Thus, it is unnecessary for the performance criteria to reflect absolute system "size."

The number of parameters required to specify channel characteristics may also be reduced by appropriate normalization procedures. It is apparent that for a random-multipath channel, just as for a channel with purely additive disturbances, the received-signal power required to accomplish a given communication task is directly proportional to the power level of the additive noise, when other factors remain constant. Thus, if S is expressed in units proportional to additive noise power, it becomes unnecessary to specify absolute power levels when evaluating and comparing different techniques. Since additive-noise power-spectral density n_o is often used to characterize the power level of a stochastic additive disturbance, it is appropriate to evaluate performance in terms of a power-requirement factor $F_P \triangleq S/n_oR$ and a bandwidth-requirement factor $F_W \triangleq W/R$, both of which are dimensionless performance criteria. An alternate power-requirement factor $\mathfrak{F}_P \triangleq S/n_o \mathfrak{R}$ may be used where it is desired to express performance in terms of maximum information-encoding rate and error rate rather than merely in terms of communication rate.

In addition to evaluating performance factors F_P and F_W, it may often be desirable to indicate the ratio of received signal power to additive noise power in the occupied band of frequencies. This is particularly easy, given values for the above criteria, for

$$\mathrm{SNR} \triangleq \frac{S}{n_o W} = \left(\frac{S}{n_o R}\right) \div \left(\frac{W}{R}\right) = \frac{F_P}{F_W} \tag{2.9}$$

Curves expressing the relations between F_P and such other system parameters as SNR, F_W, and ϵ are useful for illustrating the manner in which these factors influence the power economy of the system, and are also useful for comparing the potential advantages of unlike communication techniques.

III. ADAPTIVE MATCHED-FILTER RECEIVING TECHNIQUES

A. DESCRIPTION OF THE SYSTEM

A convenient representation for the time-varying linear filter in the random-multipath-channel model of Fig. 1 is the tapped delay line shown in Fig. 2.

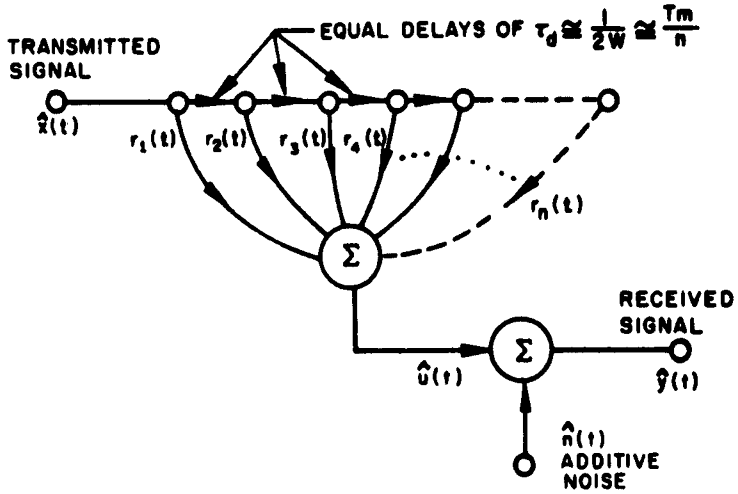


FIG. 2. MULTIPATH-CHANNEL MODEL EMPLOYING TAPPED DELAY-LINE FILTER.

It is generally possible to represent an arbitrary linear filtering operation as closely as desired with a discrete filter such as is shown in Fig. 2. [See Ref. 7.] If the signals to be processed are essentially bandlimited to W cps, a tap spacing of 1/2W sec is adequate to represent the effects of an arbitrary filter response. Time variations in the multipath characteristic are accommodated by allowing the $r_i(t)$ to be random functions of time. In the case of typical multipath channels, the nature of the sampled-data processes that underlie the tapped delay-line representation make both positive and negative polarities possible for the $r_i(t)$, a consequence that must be reflected if an analysis is to be of practical applicability.

Optimum detection of one of a set of possible signals $\{u^{(j)}(t)\}$ in additive independent white gaussian noise $n(t)$ is a well-understood process which may be based on mean-square-error comparison of $y(t)$ with each possible $u^{(j)}(t)$. For other types of additive noise the procedure is generally quite similar, but may involve special processing of y and each $u^{(j)}(t)$ prior to the comparison, in accordance with available statistical knowledge concerning $n(t)$.

In the practical case of interest here, the set of possible symbols $\{x^{(j)}(t)\}$ can be known precisely to the receiver a priori, but the $\{r_i(t)\}$ can be determined only approximately because of the effects of additive noise on any measurement process that may be used. The symbols $\{u^{(j)}(t)\}$ required for ideal detection of the received signal are unknown; they may be obtained only approximately, by operating on each $x(t)$ in accordance with obtainable information about the functions $r_i(t)$. The corresponding detection scheme is not necessarily optimum except in the limit as channel-measurement errors approach zero. The generally optimum receiving procedure appears to involve an unacceptable amount of computational labor, as described in Chapter II. While Kailath [Ref. 3] has derived some receiver forms which are optimum under the special conditions he sets forth and which appear to involve a reasonable degree of equipment complexity, it is not evident that they would be superior to the techniques considered here when the limitations of practical multipath channels are imposed.

The general philosophy of the adaptive matched-filter receiver has been developed and presented by Price and Green [Ref. 8], and an ingenious experimental binary version has been built and successfully operated. This system, which involves digital frequency-shift modulation of a wideband channel-sounding signal, is inherently unsuited to signalling alphabets much larger than binary. In order to explore the future potentials of adaptive matched-filter receiving techniques, consideration will be given here to a more general version involving the transmission of a sum of independent channel-sounding and message-carrying signals.

The proposed channel-sounding signal is essentially that suggested in Price's [Ref. 1] original work--a set of discrete sinusoidal signals uniformly spaced throughout W, the occupied band of frequencies. Though the autocorrelation function of the composite sounding signal repeats itself periodically, ambiguity in channel impulse-response measurements is avoided by making the spacing of the sine waves equal to or less than the reciprocal of T_m, the maximum expected differential multipath-propagation delay. Where a reasonably large number of these sinusoidal signals are involved, such a composite sounding signal has first-order statistics quite similar to those of bandlimited white gaussian noise of corresponding variance.

In order to prevent interaction between sounding signals and message-bearing signals, a finite bandwidth equal to B_s (the maximum expected differential doppler frequency spreading of the channel) must be reserved for exclusive occupancy by each sounding-signal component. While such a measure is not absolutely necessary, it is very desirable under many operating conditions and generally leads to more satisfactory analytical results. The total bandwidth so reserved must be at least equal to $T_m W B_s$.

Additional conditions applicable to the channel model of Fig. 2 and to the following analysis are enumerated below:

1. The message component of the transmitted bandpass signal is zero-mean and has maximum entropy within the constraints of specified frequency-spectral occupancy and variance σ_x^2, or S', equal to a constant k times the mean power S of the entire transmitted signal.
2. Additive noise is zero-mean, stationary, gaussian, white within bandwidth W, and of known power spectral density n_o.
3. No single delay-line tap multiplier dominates in its contribution to total received-signal power.
4. The various arbitrary scale factors of the system are chosen such that $\sigma_x^2 = \sigma_u^2 = S' = kS$, where $x(t)$ and $u(t)$ are the message-bearing components of the total signals $\hat{x}(t)$ and $\hat{u}(t)$.

A radio propagation scientist is likely to study and predict such items as mean received-signal power, doppler-spreading bandwidth B_s, differential propagation delay T_m, and additive-noise level n_o, given a transmitter power S and some particular type of multipath-propagation circuit. Presumably these parameters will tend to remain reasonably constant over periods of time long enough to permit useful communication with some particular system design. The radioscientist may also be interested in the number and spread of predominantly distinguishable multipath-propagation modes and in their detailed statistical structure, but this latter information is less likely to be readily predictable or available to a communication-system designer. In the following analyses an endeavor is made to characterize random multipath-channel behavior in terms of the same parameters that are likely to be studied and reported by the radio propagation scientist.

The random physical processes that affect a multipath channel are rarely stationary, but gross variations in the statistical description of the channel are likely to occur very slowly compared to information transmission rates. It is not unreasonable to assume that such random processes are stationary and ergodic over some finite time interval during which some particular system design will be used. A finite amount of information concerning the statistical state of the channel will have to be made available to the transmitter and receiver, prior to such a time interval, in order that the appropriate system "design" may be selected. Because of the simplicity of the statistical descriptions employed in the analyses of this report, it is unlikely that provision of the necessary description will detract appreciably from the performance of the system.

The exploring of communication rate R, for a message signal of maximum entropy as proposed in this chapter, implies a coding procedure involving an infinite number of infinitely enduring message symbols, a procedure philosophically similar to that suggested by Shannon [Ref. 5] for errorless transmission of information at a rate approaching channel capacity. The practical problem of intersymbol influence in multipath channels may be avoided simply by discarding the proportion of a received symbol that overlaps with its neighbor due to differential propagation-time dispersion. In a practical situation, where it may be desirable to limit symbol size and duration, such an exclusion may be of significant proportions. In such an event the special measures discussed in Appendix A are appropriate. With the suggested techniques incorporated into the system, it appears that the analytical procedures to be employed in the following pages remain approximately valid, and that the adverse consequences of a limited symbol alphabet can be handled in a manner similar to that used for a channel where random multipath is not present--for example, see Rice [Ref. 9].

B. COMMUNICATION FUNCTION OF THE SYSTEM

In order to reduce unnecessary complication, consider first the communication function of the system as though the sounding-signal components were not present and the message-signal components occupied the entirety of a fictitious bandwidth $W' = W(1 - T_m B_s)$. The rate of communication of information from the transmitter to the receiver is given by

$$R = H(y) - H_x(y) \tag{3.1}$$

where both entropy determinations are in fact dependent on, and hence conditional upon, the knowledge of the channel gained from the separate sounding processes in addition to any long-term statistical knowledge of the channel that is available. The adaptive matched-filter receiver does not necessarily use this knowledge for probability computation in an optimum sense. Such a receiver, in effect, makes an estimate of the $u^{(j)}(t)$ that would result from transmission of each of the m possible symbol choices. Each $u_{est}^{(j)}$ is then compared with the received signal y, in order to make a guess as to which $x^{(j)}$ was transmitted. All information pertaining to the $r_i(t)$ that is not preserved in the set $\left\{u_{est}^{(j)}(t)\right\}$; $j = 1, 2, 3 \ldots m$ is effectively discarded, and has no influence in the appropriate determination of the entropies of Eq. (3.1). These limitations may be partially represented by rewriting Eq. (3.1) in the following manner:*

$$R = H_{\left\{u_{est}^{(j)}\right\}}(y) - H_{x,\left\{u_{est}^{(j)}\right\}}(y) \; ; \; j = 1, 2, 3, \ldots m \tag{3.2}$$

In attempting to evaluate Eq. (3.2) analytically in terms of measurable propagation-circuit parameters, one may invoke the bounds on entropies of added independent functions derived by Shannon. [See

*See Appendix B for general proof of the validity of Eq. (3.2).

Ref. 4.] Thus, if entropy power $\bar{S}(y)$ is defined such that $H(y) = \log\sqrt{2\pi e\bar{S}(y)}$, then

$$\bar{S}(u) + \bar{S}(n) \leq \bar{S}(y) \leq S(u) + S(n) \tag{3.3}$$

The Rayleigh-fading statistics that characterize the rapid fading on most common random multipath circuits may be used to determine bounds on the entropy $H_{\left\{u_{est}^{(j)}\right\}}(u)$ and, hence, on the corresponding conditional entropy power

$$\bar{S}(u)\Big|\left\{u_{est}^{(j)}\right\}.$$

If such a result* is introduced, the following bounds are readily obtained:

$$0.316\, S' + n_o W' \leq \bar{S}(y)\Big|_{\left\{u_{est}^{(j)}\right\}} \leq S' + n_o W' \tag{3.4}$$

Bounds for the conditional entropy power corresponding to the last term of Eq. (3.2) may be expressed in a similar manner. Thus,

$$\bar{S}(u)\Big|_{x,\left\{u_{est}^{(j)}\right\}} + \bar{S}(n) \leq \bar{S}(y)\Big|_{x,\left\{u_{est}^{(j)}\right\}} \leq S(u)\Big|_{x,\left\{u_{est}^{(j)}\right\}} + S(n) \tag{3.5}$$

The conditional power

$$S(u)\Big|_{x,\left\{u_{est}^{(j)}\right\}}$$

is simply the variance of the difference between $u(t)$ and the least-integral-square-error estimate thereof that can be constructed given $x(t)$ and available information. If each $u_{est}^{(j)}$ is selected in a manner that minimizes estimation-error variance for the given channel information, then the desired conditional power corresponds to the expected value of

$$\left[u^{(j)} - u_{est}^{(j)}\right]^2.$$

*Derived in Appendix C.

The practical consequences of using a suboptimum procedure for obtaining the $\{u_{est}^{(j)}\}$ may be readily reflected in the evaluation of R. The receiver employs the estimated functions in exactly the same manner whether they are optimum or not; it has no way of either knowing or taking advantage of the fact that a better estimate could have been made. Therefore, the appropriate value to be used for

$$S(u)\Big|_{x,\{u_{est}^{(j)}\}}$$

is the error variance of the actual set of functions $\{u_{est}^{(j)}\}$ that are provided.

The functions $\{u_{est}^{(j)}\}$ may be obtained by operating individually on each of the a priori-known $\{x^{(j)}\}$ with identical adaptive channel-compensation filters derived from knowledge of the measured functions $\{r_{i_{meas}}\}$. The estimation filter may take the form of a tapped delay line, similar to that shown in Fig. 2, with tap multipliers consisting of a measured set of $r_i(t)$ that have been modified in accordance with certain auxiliary weighting functions.

The auxiliary weighting functions are required because an operation on $x(t)$ with a delay-line filter whose taps are least-integral-square-error estimates of the $\{r_i(t)\}$ does not yield a reasonably good estimate of $u(t)$. A better procedure is to multiply each $r_{i_{meas}}(t)$ with a corresponding weighting function a_i chosen to minimize an estimation-error function $e_i(t)$, which has the form

$$e_i(t) = x(t)\left[r_i(t) - a_i r_{i_{meas}}(t)\right] \tag{3.6}$$

Obviously, a_i may be an instantaneous function of $r_{i_{meas}}(t)$ and, thus, may be a time-varying function.* Assuming only that the average correlation of the error contributions of different taps is zero, the conditional variance of u is given by

*Refer to Appendix D for additional consideration of auxiliary weighting functions.

$$S(u)\Big|_{x,\left\{u_{est}^{(j)}\right\}} = S(u)\Big|_{u_{est}^{(j)}} = \sum_{i=1}^{n}\iint e_i^2(r_i, r_{i_{meas}})\, p(r_i, r_{i_{meas}})\, dr_i\, dr_{i_{meas}} \tag{3.7}$$

For the specified conditions, $r_{i_{meas}}(t)$ is composed of $r_i(t)$ plus independent gaussian noise. In order to find the optimum weighting functions and to evaluate the conditional variance of $u(t)$, it is necessary to specify the particular joint, finite, probability distribution pertaining to the sample values of the functions $r_i(t)$ which are to be measured. In general,

$$S(u)\Big|_{x,\left\{u_{est}^{(j)}\right\}}$$

will be an inverse function of the amount of detailed structure that characterizes a given multipath-propagation circuit--for example, it might be appreciably smaller for an ionospheric multipath circuit with several distinct modes than for a more uniformly time-dispersive scatter-multipath circuit under conditions that were otherwise similar.

The application of procedures for extracting and exploiting the higher-order statistics of multipath structure in generating weighting functions $\left\{a_i(t)\right\}$ goes somewhat beyond present philosophy and practice with regard to adaptive matched-filter multipath receiving techniques. It is quite possible that there will be few applications which will justify the use of weighting functions based on anything more complicated than a characteristic first-order long-term probability distribution of tap-sample values for the general classification of multipath channel involved. Note that the implementation of a system based on a simplified statistical description of channel-model parameters carries no impractical connotations; although, in general, the performance of such a system may deteriorate when it is actually used on a channel with characteristics other than those assumed.

For many types of scatter-multipath channels the $\left\{r_j(t)\right\}$ appear to be independent, gaussian, and zero mean. For ionospheric multipath channels, one might expect a similarly shaped first-order probability distribution, but perhaps with a sizable impulse at $r_i(t) = 0$ that would influence the a_i generation process considerably.

In the following pages consideration will be given to the performance of an adaptive matched-filter receiver design based on a limited optimization of the a_i for the case where the $r_i(t)$ are assumed to be equal-variance, gaussian, and independent. Refer to Appendix D for specific details of the optimization procedure involved. As will be shown subsequently, the derived performance bounds for this receiver are the same in connection with any other particular set of r_i probability statistics as they are for a channel actually having the detailed r_i statistics assumed in Appendix D. The drawback is that such a receiver will not work as well on other types of channels as would a design optimized for their specific individual probability statistics. Reasons for giving detailed consideration to the particular design proposed in Appendix D are as follows:

1. Within the specified design latitude, the resulting weighting functions appear to be optimum for many types of scatter-multipath channels.
2. The system implementation is particularly simple.
3. The resulting performance calculations for this system are valid when it is used, without any design modification, on practical multipath channels with a wide range of tap multiplier statistics.

In connection with reason 2, it will subsequently be seen that under some operating conditions there is only a very limited theoretical potential for performance improvement associated with greater freedom to optimize the a_i for particular types of channels.

Since W' will always be somewhat less than W, adjacent samples of $x(t)$ spaced $1/2W$ sec apart in time will not be entirely independent. In obtaining the variance of $u(t)$, however, there is good reason for simply assuming that message-signal components arriving at the receiver via the different delay-line model taps are uncorrelated. In the first place, for most well behaved multipath channels $W' \approx W$, so that adjacent samples of $x(t)$ are indeed very nearly independent. In the second place, it is unlikely that there will be much, if any, long-term correlation in the random values of adjacent delay-line-tap multiplier functions $\{r_i(t)\}$. Even in the case of ionospheric multipath circuits, the dependence of adjacent members of the $\{r_i(t)\}$ will probably appear as correlation in magnitudes rather than in the signed values of the multiplier functions. Therefore, it appears that the variance of $u(t)$ will be expressed accurately by

$$\sigma_u^2 = \sum_{i=1}^{n} \sigma_{r_i}^2 \sigma_x^2 = n\sigma_r^2\sigma_x^2 = n\sigma_r^2\sigma_u^2 \tag{3.8}$$

so that $\sigma_r^2 = 1/n$. By defining a mean delay-line tap measurement SNR,

$$\gamma \triangleq \frac{\sigma_r^2}{\sigma_{r_{error}}^2} \tag{3.9}$$

one may obtain

$$\sigma_{r_{error}}^2 = \frac{1}{n\gamma} \tag{3.10}$$

where $r_{i_{error}}(t)$ is equal to $\left[r_{i_{meas}}(t) - r_i(t)\right]$, a low-pass gaussian noise (due to the near gaussian, independent nature of total additive noise in the measurement process to be considered in Section III-D). The desired auxiliary weighting functions, derived in Appendix D for the assumed conditions, are simply $a_i(t) = \frac{\gamma}{1+\gamma}$ for all i and all t. The resulting error variance in $u_{est}(t)$ is also found in Appendix D to be

$$\sigma_{u_{error}}^2 = S(u)\Big|_{x,\left\{u_{est}^{(j)}\right\}} = \frac{\sigma_u^2}{1+\gamma} = \frac{kS}{1+\gamma} \tag{3.11}$$

The problems of obtaining a valid analytical expression for the conditional entropy power

$$\bar{S}(u)\Big|_{x,\left\{u_{est}\right\}}$$

are more difficult than for

$$S(u)\Big|_{x,\left\{u_{est}\right\}}$$

because of the complex nature of the composition of the error in $u_{est}(t)$. Fortunately, it is not necessary to precisely evaluate the conditional entropy power. By recognizing the limitations of the adaptive matched-filter detection technique and taking advantage of certain properties of added stochastic signals, one may greatly simplify the task.

If a system is to approach the rate of communication indicated in Eq. (3.2), it is necessary that the receiver detection process reflect optimum probability computation, based on all the useful knowledge contained in the set of waveforms $\left\{u_{est}^{(j)}(t)\right\}$. Such a detector must take advantage of special estimation-error statistics which tend to reduce the entropy power to less than that of a random white gaussian noise of equal variance. It is shown in Appendixes E and F, however, that the integral-squared-error-detection process of the adaptive matched filter has certain limitations that do not permit full advantage to be taken of the constrained statistics of the estimation error. There is additional reason, given in Appendix F, why the performance limitation should be given quite accurately by calculations based on the assumption that the estimation error is a random white gaussian variable. Thus, for the purposes of this investigation, it is appropriate to set

$$\bar{S}(u)\Big|_{x,\left\{u_{est}\right\}}$$

equal to

$$S(u)\Big|_{x,\left\{u_{est}\right\}}$$

Introducing this result into Eq. (3.5), and noting that the power and entropy power of the additive noise of the channel must also be equal, one readily obtains

$$\bar{S}(y)\Big|_{x,\left\{u_{est}^{(j)}\right\}} = \frac{S'}{1+\gamma} + n_oW' = \frac{kS}{1+\gamma} + n_oW' \tag{3.12}$$

and hence,

$$H_{x,\left\{u_{est}^{(j)}\right\}}(y) = \log\sqrt{2\pi e\left(\frac{kS}{1+\gamma} + n_oW'\right)} \tag{3.13}$$

Similarly, from Eq. (3.4) one obtains the bounds

$$\log \sqrt{2\pi e(0.316kS + n_o W')} \leq H_{\{u_{est}^{(j)}\}}(y) \leq \log \sqrt{2\pi e(kS + n_o W')} \qquad (3.14)$$

If the foregoing entropy values per sample point are multiplied by the message-signal dimensionality $2W(1 - T_m B_s)$ per unit time, and then substituted into Eq. (3.2), the following bounds for the rate of communication of the system are obtained:

$$(1-T_m B_s)W \log\left[\frac{0.316kS + n_o W(1-T_m B_s)}{\frac{kS}{1+\gamma} + n_o W(1-T_m B_s)}\right] \leq R \leq (1-T_m B_s)W \log\left[\frac{kS + n_o W(1-T_m B_s)}{\frac{kS}{1+\gamma} + n_o W(1-T_m B_s)}\right]$$

(3.15)

At this point it is appropriate to verify a previously stated conclusion concerning the rate of communication achieved on channels that do not have the assumed statistical characteristics. As long as the narrowband Rayleigh statistics generally associated with rapid multipath fading persist, then the bounds on $H_{\{u_{est}\}}(y)$ that have been derived in Appendix C are in no way affected by the details of the statistics of the $r_i(t)$. The error variance of the estimate of $u(t)$, which determines the proper expression to be used for $H_{x,\{u_{est}\}}(y)$, is a function only of the variance of $x(t)$ and the variance of the error in the measurement of the $r_i(t)$, since the a_i used in the proposed system were constant. The noise arising in the measurement of the i^{th} tap multiplier of the delay line is, to a very good approximation, independent of the mean value of that measurement, which is r_i, and of the value of any other tap. Thus, $H_{x,\{u_{est}\}}(y)$ is independent of the r_i and, hence, of any particular joint-probability distribution that they may have. One must conclude, then, within the mildly restrictive limitations specified at the outset of this section, that the derived bounds on R for the proposed system design are valid for any statistical distribution of the $\{r_i(t)\}$.

C. SOUNDING FUNCTION OF THE SYSTEM

A cross-correlation operation may be used to determine the tap-multiplier values of the delay-line channel simulator to be used in estimating the $\left\{u^{(j)}(t)\right\}$. It is now convenient and proper to ignore received-signal message-bearing components, which do not influence the measurements. Let the signal $y(t)$, equal to the received sounding signals plus noise, be multiplied by a stored reference version of the composite sounding signal, aligned in delay with the received signal arriving via the i^{th} delay-line tap. The result is

$$\left[x_s(t-T_S-i\tau_d)\right]y(t) = \left[x_s(t-T_S-i\tau_d)\right]\left[\hat{n}(t) + \sum_{j=1}^{n} r_j(t)x_s(t-T_S-j\tau_d)\right] \quad (3.16)$$

where T_S is the propagation delay corresponding to the shortest path, and τ_d is the delay-line-tap spacing. Received-signal contributions differing in delay by integral multiples of τ_d from the i^{th} one are poorly correlated with $x(t - T_S - i\tau_d)$ and, thus, have essentially the same influence on the resulting measurement as would corresponding amounts of random noise. The total variance of uncorrelated sounding signals and added noise at the i^{th} tap is given by

$$\sigma^2_{n_{eff}} = n_oW + \sum_{\substack{j=1 \\ j\neq i}}^{n} \sigma^2_{r_j}\sigma^2_{x_s} \cong (1 - k)S + n_oW \quad (3.17)$$

The above approximation is good under the assumed condition that signal contributions of individual taps are relatively small. Equation (3.16) may be rewritten in the following manner:

$$\left[x_s(t - T_S - i\tau_d)\right]y(t) = \left[x_s(t')\right]\left[r_i(t)x_s(t') + n_{eff}(t)\right] \quad (3.18)$$

Since $r_i(t)$ will normally vary much more slowly than $x_s(t)$, in a brief measurement time interval $r_i(t)$ may be regarded as constant. Because $n_{eff}(t)$ is essentially uncorrelated with $x_s(t)$, the expected value of the cross-correlation product of Eq. (3.18) is simply $r_i\sigma^2_{x_s}$, equal to $(1 - k)r_1S$. If this product is divided by [illegible])S, the result may be represented by the equation

$$(r_1)_{meas} = r_1 + \frac{x_s(t')n_{eff}(t')}{(1 - k)S} + \text{intrinsic measurement noise} \quad (3.19)$$

In Eq. (3.19), the intrinsic noise term arises because x_s^2 is equal to $(1 - k)S$ only at particular instants of time. However, since $x_s(t')$ is an <u>a priori</u>-known function, the effects of the intrinsic measurement variability may be eliminated, so that primary attention may be given to the measurement noise contribution represented by the next-to-last term of Eq. (3.19). Since $\sigma_{x_s}^2$ is equal to $(1 - k)S$, the variance of the total effective measurement noise is given by

$$\sigma_{noise}^2 = \frac{\sigma_{n_{eff}}^2}{(1 - k)S} = \frac{(1 - k)S + n_oW}{(1 - k)S} \quad (3.20)$$

The power spectrum of the noise in the cross-correlation product, obtained by convolving the discrete sounding-signal spectrum with its doppler-spread, noisy, received replica, has a rather weird fine structure that can be adequately represented as shown in Fig. 3.

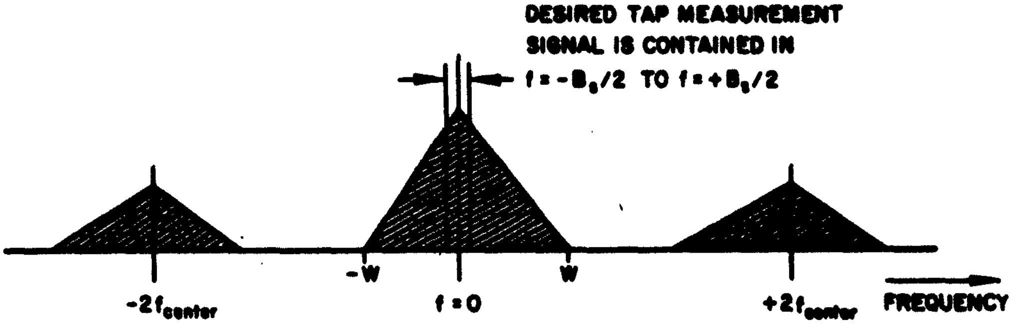


FIG. 3. SPECTRUM OF NOISE IN TAP-MEASUREMENT PROCESS.

A low-pass filter having a minimum bandwidth of $B_s/2$ can be used to reject excess measurement noise. Half of the total measurement noise power appears in the region of the frequency origin in Fig. 3. Of this half, the fraction

$$\left[\int_0^{B_s/2}(W - f)df\right] \div \left[\int_0^{W}(W - f)df\right]$$

equal to $(B_s/W)[1 - (B_s/2W)] \approx B_s/W$ will be accepted by the measurement filter. Thus, the final variance of the measurement error is given by

$$\sigma^2_{r_{error}} = \frac{(1 - k)S + n_oW}{(1 - k)S}\left(\frac{B_s}{2W}\right) \tag{3.21}$$

Using Eq. (3.21) and the relation $\sigma^2_{r_1} = 1/n = \frac{1}{2T_mW}$, the expression

$$\gamma \triangleq \frac{\sigma^2_{r_1}}{\sigma^2_{r_{error}}} = \frac{(1 - k)S}{T_mB_s[(1 - k)S + n_oW]} \tag{3.22}$$

may be readily obtained for the tap-measurement signal-to-noise ratio.

D. OVERALL PERFORMANCE OF THE SYSTEM

The value of γ from Eq. (3.22) may be substituted into Eq. (3.15) in order to obtain bounds on R as a function of W, S, n_o, k, T_m, and B_s. To investigate the nature of these results, it is convenient to substitute $SNR = S/n_oW$ and to express the result as bounds on $F_p \triangleq S/n_oR = \frac{(W)(SNR)}{R}$. Thus

$$\frac{SNR}{(1 - T_mB_s)\log_2\left\{\dfrac{k(SNR) + (1 - T_mB_s)}{\dfrac{k(SNR)}{1 + \dfrac{(1 - k)SNR}{T_mB_s\left[(1 - k)SNR + 1\right]}} + (1 - T_mB_s)}\right\}} \geq F_p \geq \frac{SNR}{(1 - T_mB_s)\log_2\left\{\dfrac{0.316\,k(SNR) + (1 - T_mB_s)}{\dfrac{k(SNR)}{1 + \dfrac{(1 - k)SNR}{T_mB_s\left[(1 - k)SNR + 1\right]}} + (1 - T_mB_s)}\right\}} \tag{3.23}$$

While the denominator of the upper bound of Eq. (3.23) can become negative, at such times an alternate limit of $F_p \leq \infty$, based on the fundamental principle $H_x(y) \leq H(y)$, is appropriately applied to get a sensible result. A typical example of the behavior of these bounds as k is varied is shown in Fig. 4 for specified system constants:

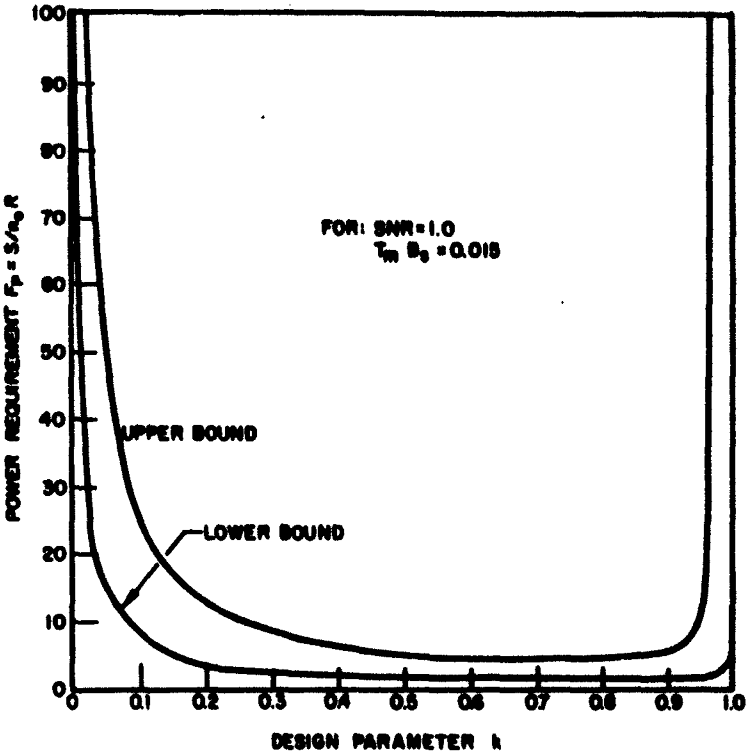


FIG. 4. POWER-REQUIREMENT FACTOR F_p VS k.

For investigating the nature of system performance bounds as the channel characteristics and the SNR are varied, a large number of curves similar to those of Fig. 4 have been calculated and plotted. The following interesting observations have been made in this process:

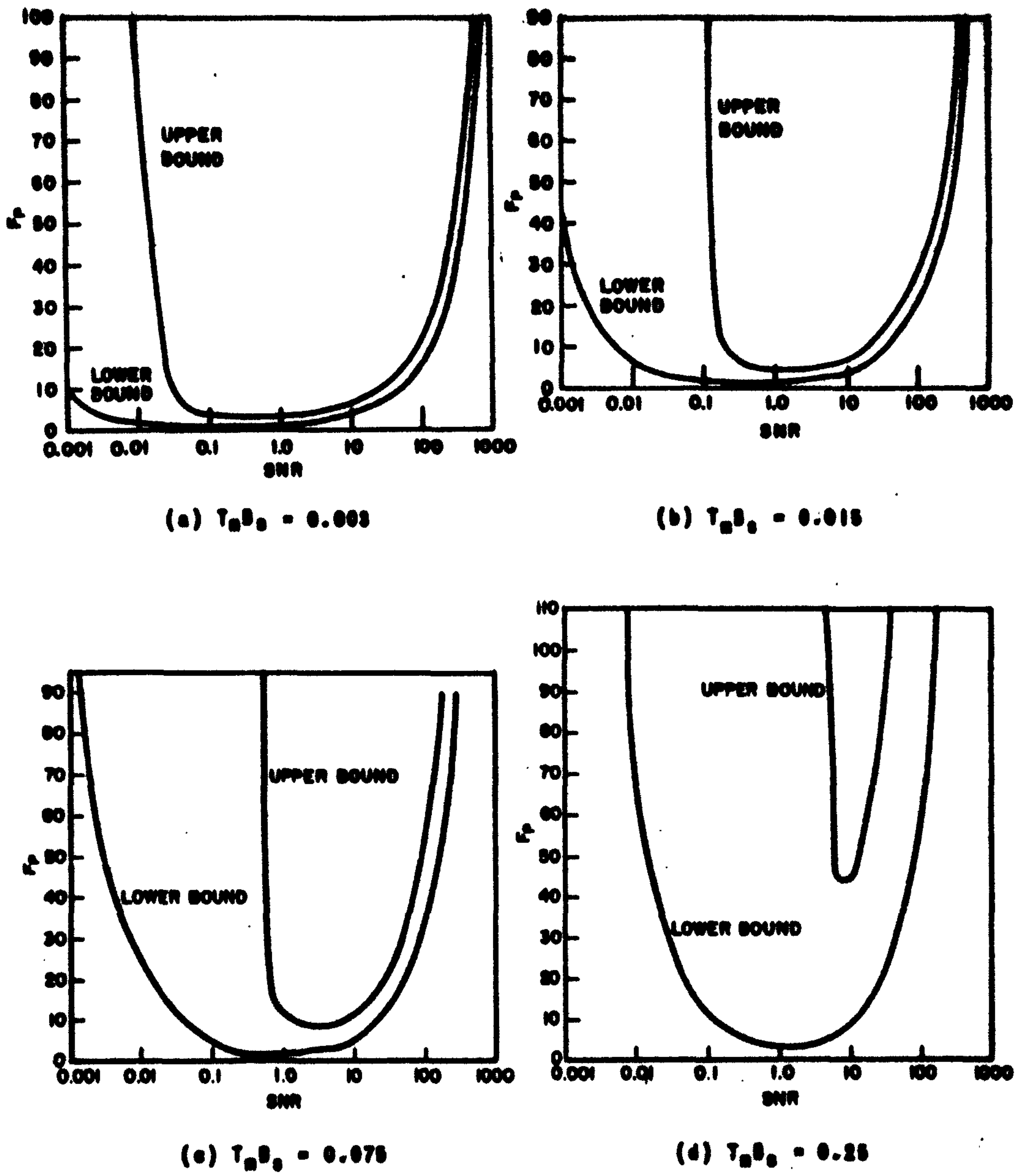
1. In most cases, the performance bounds are flat over surprisingly wide ranges of k adjacent to the minima. The decrease in message power resulting from greater concentration on the channel-sounding function appears to be almost exactly compensated by the improved channel knowledge in the vicinity of the minima.
2. As the product $T_m B_s$ increases (between zero and one), the optimum values of F_p occur for smaller values of k, indicating that a

larger proportionate effort should be devoted to the channel-sounding function if an adaptive matched-filter technique is to be used. Similarly, the minima occur at larger values of F_p, indicating the increasing degradation caused by the channel.

3. There are cases where the bounds exhibit very slight but distinct double minima as k is varied. In addition, the minima for the upper and lower bounds, in any given case, may occur for values of k that are quite widely separated. Neither of these observations appears to be of much practical significance, however, in view of the flatness of the bottoms of the curves.

In Fig. 5, the minimum points from Fig. 4 and numerous other similar sets of curves are plotted as functions of SNR for various values of the product $T_m B_s$. Notice that the choice of receiver-operating SNR appears to become much more critical as $T_m B_s$ increases.

Similar sets of curves have been determined [see Ref. 10] with an allowance made for a nonideal tap-measurement filter. The results are quite similar, except that the adverse effects of an increase in $T_m B_s$ become evident much sooner. At the present time, it appears that advances in delay-line technology will soon allow near-ideal tap-measurement filters to be employed. For this reason, no allowance for nonideal filters has been made in the present calculations.



(a) T_mB_s = 0.003

(b) T_mB_s = 0.015

(c) T_mB_s = 0.075

(d) T_mB_s = 0.25

FIG. 5. BOUNDS ON POWER-REQUIREMENT FACTOR F_p.

IV. INCOHERENT RECEIVING TECHNIQUES

A. PROPAGATION CIRCUITS REQUIRING SPECIAL TREATMENT

Recently there has been increasing interest in radio propagation modes involving scattering from particles with large differential velocities. It appears possible that, in the future, highly reliable, long distance communication circuits may be based on signal scattering from waves of electrons present in the plasma of the F-region ionosphere [Refs. 10 - 13]. The scattering of signals from belts of resonant dipole chaff, placed in orbit around the earth, offers similar possibilities [Ref. 14]. The differential velocities of such scattering particles and the associated spatial distribution of the scattering phenomena are likely to result in large values of the product $T_m B_s$.

It has been found that most communication techniques, including the adaptive matched-filter techniques considered in Chapter III of this report, cease to function well as $T_m B_s$ increases toward and beyond unity. The received signals become much like independent random noise, and it may be difficult to learn very much about the specific waveshape of a transmitted signal from observations made of the received signal. Coherent and semicoherent detection procedures, which require a reasonable *a priori* receiver knowledge of possible received-signal waveshapes, are not appropriate under the circumstances described.

B. DESCRIPTION OF PROPOSED SYSTEM

Regardless of the nature of the propagation circuit, a cause-and-effect relation still exists between the energy of a narrowband pulse transmission and the received energy observed in a corresponding time interval and doppler-spread frequency band. A promising communication procedure, for use on circuits with large $T_m B_s$ products, involves sinusoidal pulse transmission with simple frequency-shift or time-position-shift modulation, or a combination of these techniques. The size of the shifts should be selected so that the received waveforms, corresponding to different possible transmitted pulses, can be observed independently. Some examples of received signals for various modulation schemes are indicated in Fig. 6.

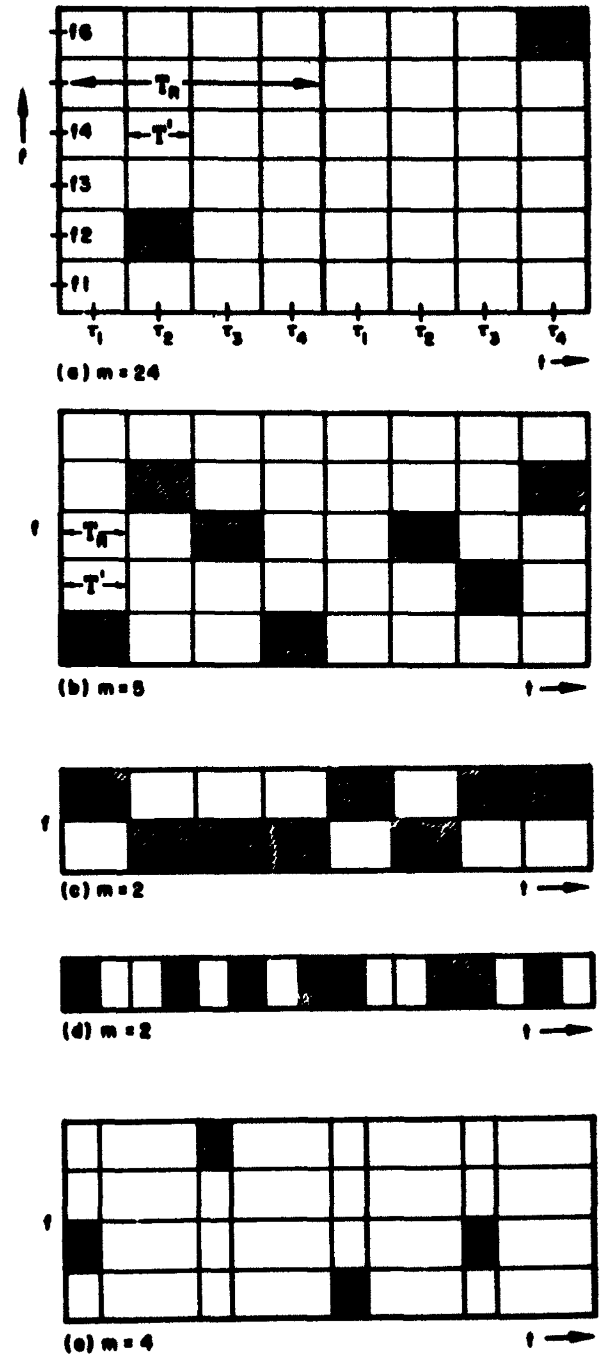


FIG. 6. EXAMPLES OF MODULATION SCHEMES.

In Fig. 6, m is the number of possible positions per pulse, equal to the digital alphabet size, and T_R is the symbol repetition time. To avoid signal overlap, T' must exceed the transmitted pulse duration by T_m; and Δf, equal to $f_{n+1} - f_n$, must exceed the transmitted pulse bandwidth by B_s.

Price [Ref. 6] has shown that a detection scheme based on radiometric (energy-content) measurements is optimum for certain types of single-path scatter circuits. While a general treatment of more complicated channel models in an optimum manner did not appear to be mathematically tractable, Price reasons that his result should apply equally well to a wide variety of unpredictable scatter-type multipath-propagation circuits. It is therefore appropriate to base receiver decisions, in the proposed system, on simple energy measurements. There will be filtering and time-gating circuits for selecting the different possible received pulses, followed by m individual energy-measurement operations.

C. DERIVATION OF PERFORMANCE EQUATIONS

To evaluate the performance possible over these scatter channels, one needs to be able to anticipate transmission rates and error probabilities resulting from various received power levels when various possible modulation schemes are employed. The probability of a correct receiver decision corresponds to the probability that the average signal-plus-noise power observed in a transmitted-pulse location exceeds the average noise powers observed, respectively, in each of the m-1 vacant pulse locations. Present theory indicates that the rapid signal-amplitude fading of such circuits obeys Rayleigh envelope statistics, as does the relatively narrow band of additive gaussian noise frequently accompanying the received signal. Thus, the probability of a correct decision corresponds approximately to the probability that the energy observed in a Rayleigh-fading function of variance $\hat{S} + N$ and duration T exceeds the maximum energy among m-1 similar functions, each of variance N. It readily may be seen that, for a given pulse power and duration, the particular breakdown of an m into frequency steps and pulse-position steps does not influence the error probability, provided n_o is reasonably constant over the band of frequencies occupied.

It appears that a precise analysis of error probabilities for an optimum-receiver implementation, given some particular doppler-spreading characteristic, would be unnecessarily complicated and restrictive for most practical applications. In the following analysis it has been assumed merely that a received-signal pulse and its accompanying noise are gaussian and have rectangular frequency spectra of bandwidth B_s. To make a reasonable allowance for additional received-pulse bandwidth, resulting from a finite transmitted-signal bandwidth, the relation $N = n_o(B_s + 1/T)$, rather than simply $N = n_oB_s$, has been introduced. A detailed examination of the resulting performance curves has revealed that such a relation gives very good continuity with the error rate required for zero-information transmission when the opportunity to observe the symbol vanishes (that is, $T = 0$ and received energy $= 0$; see asymptotic limits in Fig. 7).

In the derivation of expressions for error rate, it is helpful to represent received signals and their energies in terms of samples taken at appropriate time intervals, as suggested by Shannon [Ref. 5]. For the conditions assumed above, samples taken at intervals $1/2B_s$ are independent, and the statistical properties of the total energy observed in an interval T are correctly described by a chi-square distribution with $n = 2TB_s$ degrees of freedom. If average received-signal-pulse power during T is designated $\hat{S}$, observed energy is proportional to a sum x of instantaneous power observations, described by

$$P_{\hat{S}+N}(x) = \frac{x^{\frac{n}{2}-1}\exp\left[-\frac{x}{2(\hat{S}+N)}\right]}{\left[2(\hat{S}+N)\right]^{n/2}\Gamma(n/2)} \tag{4.1}$$

A similar expression with variance N describes the corresponding sum of power observations when no signal pulse is present:

$$P_N(x) = \frac{x^{\frac{n}{2}-1}\exp\left(-\frac{x}{2N}\right)}{(2N)^{n/2}\Gamma(n/2)} \tag{4.2}$$

At the receiver there will be m energy-measurement circuits, with m equal to the product of the number of possible arrival times and the number of possible frequencies available to each pulse. The largest energy occurring among the $m-1$ individual noise measurements has a probability distribution corresponding to

$$P_{N_{max}}(x) = (m - 1)P_N(x)\left[\int_0^x P_N(\gamma)d\gamma\right]^{m-2} \tag{4.3}$$

For a symmetrical digital system, a receiver-decision error occurs whenever the largest noise observation exceeds an observation of signal plus noise. Thus,

$$\epsilon = \int_0^\infty \left[P_{S+N}(x) \int_x^\infty P_{N_{max}}(\beta)d\beta\right]dx \tag{4.4}$$

In principle, the error rate given by Eq. (4.4) can be evaluated analytically with the use of integral expressions tabulated in series-expansion form. For typical values of m and n, however, Eq. (4.4) breaks up into a formidably large number of such terms. It has generally been found more practicable to evaluate error rate directly from the probability-distribution expressions by means of numerical integration on a general-purpose computer.

D. RESULTS

With a symmetrical modulation system, information may be encoded for transmission at a maximum rate

$$\mathcal{R} = \frac{1}{T_R} \log_2 (m) \tag{4.5}$$

The noise power accompanying a doppler-spread received pulse has been given by

$$N = n_o\left(\frac{1}{T} + B_s\right) = \frac{n_o}{T}\left(1 + \frac{n}{2}\right) \tag{4.6}$$

Average received-signal power S is given by

$$S = \hat{S}\left(\frac{T}{T_R}\right) \tag{4.7}$$

Making use of the definition $\widehat{SNR} \triangleq \hat{S}/N$, one obtains

$$\mathfrak{F}_P \triangleq \frac{S}{n_o \mathfrak{R}} = \frac{\hat{S}T}{n_o \log_2 m} = \frac{\widehat{SNR}\,(1 + n/2)}{\log_2 m} \tag{4.8}$$

Given a value of $\widehat{SNR}$, the number n of independent signal observations per pulse, and an alphabet size m, one can determine error rate ϵ and power-requirement factor $\mathfrak{F}_P$ from Eqs. (4.4) and (4.8). There are, thus, five variable parameters involved in the calculations, of which only three can be independently specified. Such results can be presented in a large number of different ways. In Fig. 7, parametric relations between $\mathfrak{F}_P$ and ϵ are plotted for specified values of $\widehat{SNR}$ and m. Specification of n causes the plotted relations to degenerate into points, as indicated.

In Fig. 8, another method of illustrating the performance has been used. If $\mathfrak{F}_P$ is plotted as a function of $\widehat{SNR}$ for specified values of m and ϵ, the curves exhibit certain uniform characteristics that appear to be of considerable basic importance.

Each point on a curve in Fig. 8 corresponds to a particular value of n that may be readily determined from Eq. (4.8). The value of n corresponding to the minimum of a curve may be regarded as the optimum order of time diversity for corresponding values of m and ϵ. A basic mechanism involved in these techniques is, indeed, post-detection time diversity. Knowing n, the corresponding pulse duration $T = n/2B_s$ can be determined. Alternately, one may obtain an expression for T directly in terms of the explicit parameters of Fig. 8. By a suitable manipulation of Eq. (4.8), one obtains

$$T = \frac{1}{B_s}\left[\frac{\mathfrak{F}_P \log_2(m)}{\widehat{SNR}} - 1\right] \tag{4.9}$$

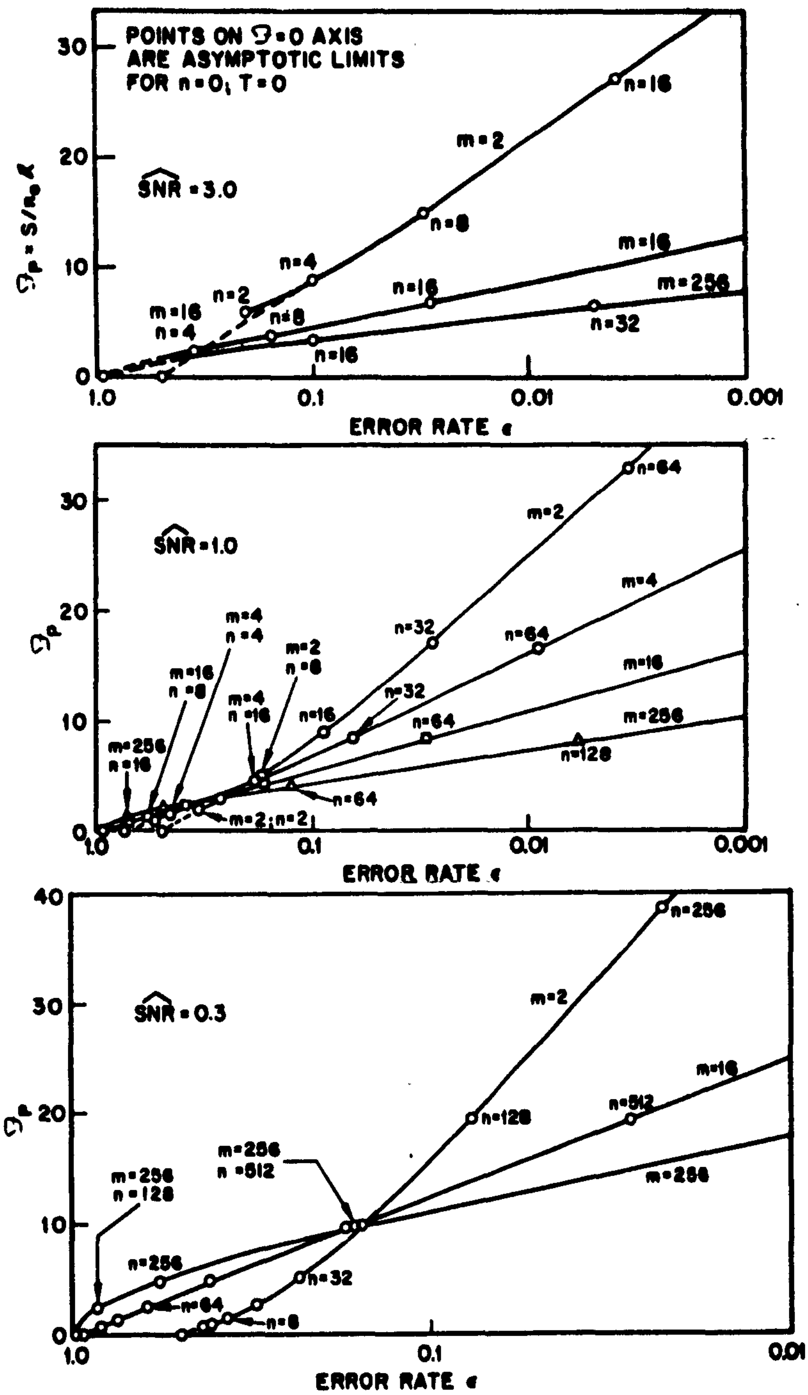


FIG. 7. $\mathfrak{F}_p$ VS ϵ FOR VARIOUS $\widehat{SNR}$.

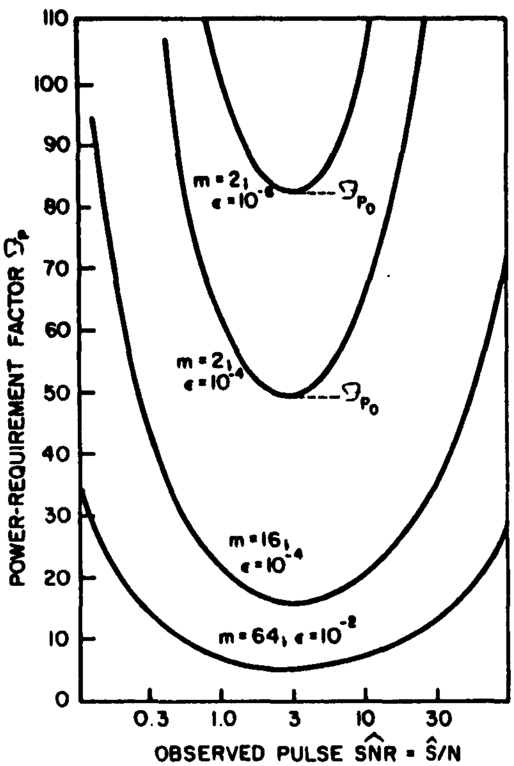


FIG. 8.

POWER-REQUIREMENT FACTOR VS $\widehat{SNR}$.

The curves of Fig. 8 exhibit a behavior that appears to be characteristic (although apparently not recognized heretofore) of a wide variety of post-detection diversity-combining techniques. These particular results were obtained, in effect, for a general m^{th}-order orthogonal-alphabet system with n^{th}-order time diversity (it would be $n/2^{th}$-order time diversity if envelope observations were made). Except for secondary pulse-time-dispersion effects, these results apply equally as well to an m^{th}-order alphabet system with q^{th}-order frequency diversity and n/q^{th}-order time diversity. The basic observation to be made about such systems is that no matter what error rate is involved, what alphabet size is used, or what combination of time and frequency diversity (and even space diversity for a fixed total antenna aperture) is used, the minimum received-energy requirement per transmitted bit occurs when the SNR of signals observed in each diversity channel is very nearly equal to three. This same result occurs implicitly in the results of a recent investigation of the optimum order of diversity for a special binary system by Pierce [Ref. 15]. Division of Pierce's "total SNR" by the "optimum number of diversity branches" yields a result very close to the magic number three in every case, even though the analytical approach to the problem is quite different from that employed here.

Another very convenient scheme can be used for presenting performance data for an extremely wide range of operating conditions. If one normalizes the ordinates of curves, such as those of Fig. 8, in terms of their respective minimum values, the curves become very nearly coincident, except in some special cases simultaneously involving high error probability and low alphabet size. Knowing this normalized characteristic curve, one needs to know additionally only the minimum (optimum) value $\mathcal{F}_{P_O}$ for the desired values of m and ϵ. The curves of Figs. 9 and 10 summarize the required information quite conveniently.

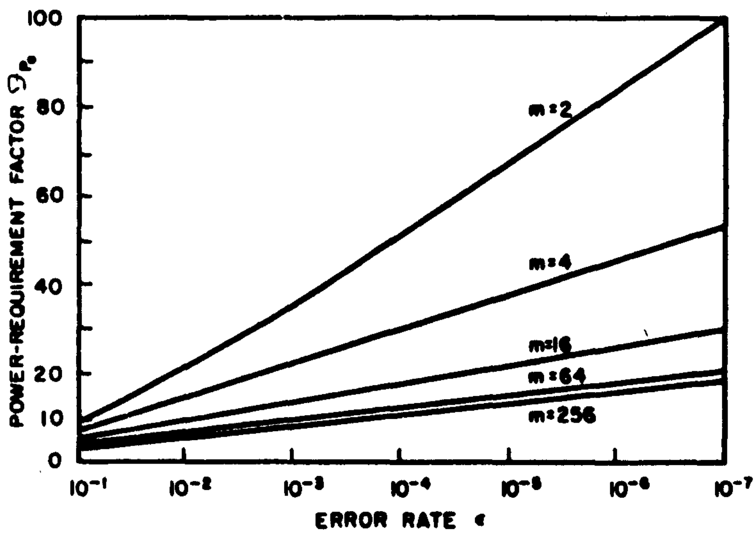


FIG. 9. MINIMUM POWER-REQUIREMENT FACTOR VS ERROR RATE.

In all cases that have been investigated, the characteristic curve of Fig. 10 has been found to represent performance at low values of $\widehat{SNR}$ quite accurately. It begins to fail for simultaneous $\epsilon > 0.1$ and

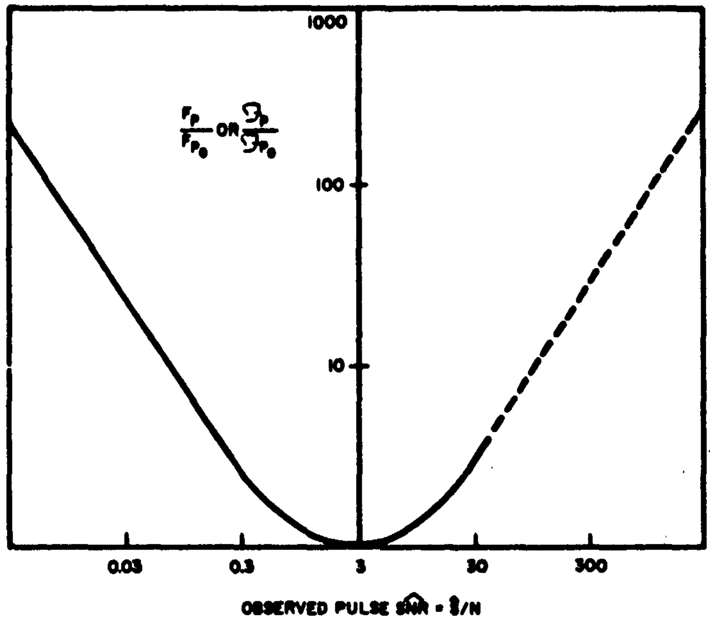


FIG. 10. NORMALIZED POWER-REQUIREMENT FACTOR VS $\widehat{SNR}$.

$m < 8$ at a $\widehat{SNR}$ of about unity, principally because the minimum shifts to a $\widehat{SNR}$ somewhat less than three in such instances.

As $\widehat{SNR}$ is increased in any given case, there is a corresponding decrease in the pulse duration required to maintain a specified error rate, and n likewise diminishes. At large values of $\widehat{SNR}$, the curve of Fig. 10 generally has been found to be accurate to within plus-or-minus one db until n decreases to a value of two, at which time the validity of the underlying assumptions is considerably strained. Analytical solutions [Ref. 16] of Eq. (4.4) for $n = 2$ and various values of m indicate that $n > 2$ for $\epsilon < 1/\widehat{SNR}$, so that one may use Fig. 10 with reasonable confidence for $\widehat{SNR}$ values less than the reciprocal of the particular error rate involved.

For comparison purposes, it may be desirable to consider the maximum rate R at which information may be encoded for errorless transmission through the system. If the entire system, including channel, transmitter, and receiver, is regarded as a discrete digital channel with alphabet size m and symmetrical error probability ϵ, then the maximum R is simply equal to the capacity of the discrete channel, and is given by Eq. (2.8).

If the curves of Fig. 9 are modified in accordance with Eq. (2.8), the results appear as shown in Fig. 11. The normalized characteristic of Fig. 10 is equally applicable whether one is concerned with energy per transmitted bit or per communicated bit.

While the results shown in Fig. 11 favor operation of the system at a high error rate, the power penalty for failure to do so is relatively mild. In practice, a very low overall error rate is frequently desired--it can be obtained either directly by operating the receiver at a low decision-error rate, or indirectly by use of error-correcting coding techniques. The present state of the art in error-correcting coding appears to be somewhat primitive insofar as achieving a low overall error rate on a digital channel having a high decision-error rate is concerned. For this reason, it may be presently more attractive to take the direct route to achieving low error rate for systems based on incoherent receiving techniques.

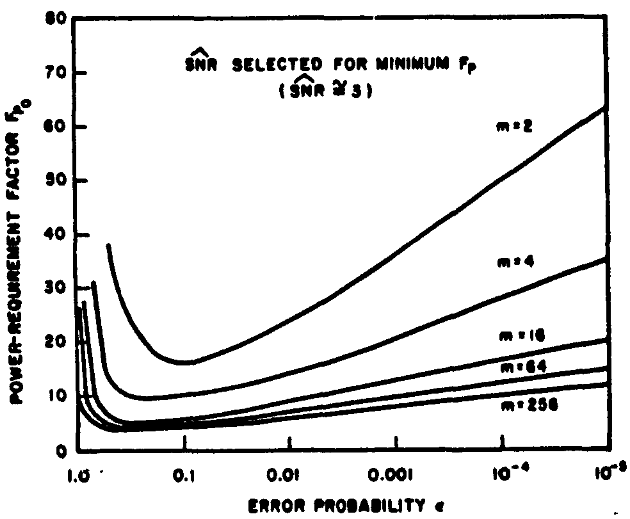


FIG. 11. MINIMUM POWER REQUIREMENT F_p VS ERROR RATE.

Intersymbol-influence problems are unlikely to cause significant performance degradation when T is appreciably greater than T_m. For the techniques that have been considered in this chapter, it is generally found that the number n of signal observations per pulse may be made quite large without sacrificing power. From the equation

$$T = \frac{n}{2B_s} = \left(\frac{n/2}{T_m B_s}\right) T_m \tag{4.10}$$

one observes that T may be made much greater than T_m even for $T_m B_s > 1$, thus effectively reducing the adverse consequences of multipath time dispersion.

E. SOME SPECIAL SYSTEM-DESIGN CONSIDERATIONS

From a standpoint of power conservation, it has been found desirable to provide a received-pulse $\widehat{SNR}$ of about three in the doppler-spread bandwidth B_s. If there is greater received-signal-pulse power available, it can be used efficiently and conveniently by dividing it equally among parallel systems operating on adjacent frequencies or sets of frequencies. These individual subsystems may carry independent message information, or they may be operated in unison to decrease T, and thus increase R, at constant n, m, ϵ, and F_p. It makes little difference, in terms of system power efficiency, which alternative is selected as long as T remains appreciably greater than T_m.

If the available bandwidth allocation is limited, one may be forced to accept the degradation indicated in Fig. 10 in order to take advantage of received-signal power appreciably greater than $n_o B_s$. The relative advantages of using a higher pulse power and consequent lower duty factor to permit the increasing of m, when total bandwidth and average received power are fixed, can be evaluated readily in particular cases. There appears to be no general rule for system optimization in such circumstances.

If one is limited to a receiving antenna of given total aperture, there may be two good reasons for avoiding predetection signal-combining procedures. In the first place, predetection combining may decrease antenna beamwidth, perhaps decreasing the common scattering volume that may be illuminated. In the second place, if total available bandwidth is limited, it may be desirable to divide the antenna into several spatially diversified segments, each with its own receiving subsystem operating nearer the optimum value of $\widehat{SNR}$.

V. COMPARISON AND APPRAISAL OF TECHNIQUES

A. FUNCTIONAL DIFFERENCES AMONG SOME MULTIPATH SYSTEMS

The principles of a number of different multipath techniques may be compared by starting with elementary systems that might be employed on channels without random multipath disturbances. If the bandwidth of the system is narrow compared to $[T_m]^{-1}$ for the channel, the salient features of random channel fluctuations are describable in terms of the time-varying attenuation and phase shift of a transmitted sine wave at the center frequency of the channel. Conventional communication systems frequently use near-sinusoidal symbols, and allow the transmitter to select certain discrete values of amplitude or phase, in successive time intervals of duration T, in order to convey intelligence. At the receiver, the signal is observed in each appropriate time interval, and an estimate of amplitude or phase, as required, of each transmitted symbol is made. If the channel transfer function changes slowly enough, an averaging of continuous received-signal observations during each T can be made by processing the received signal with a narrowband filter and sampling the output amplitude and phase once per symbol.

A necessary restriction, for successful operation in the simple manner described above, is that T be less than $1/B_s$. Such a condition assures that the useful coherent receiver-integration time will be established primarily by symbol duration, and not by the characteristics of the channel. If T is greater than $1/B_s$, the signal is simply not phase-coherent over time interval T, and a more complicated receiving procedure must be used. One may still make coherent observations of received-signal amplitude and phase, but there now will be more than one pertaining to each estimate of transmitted-symbol amplitude and phase. In order to combine observations correctly and weight them in accordance with their respective degrees of probable accuracy, the mean channel attenuation and phase during each individual observation must be known. The process of making attenuation and phase measurements in order to achieve optimum, coherent, signal detection over intervals of time longer than $1/B_s$ is, however, merely the channel-measurement procedure of the adaptive matched-filter receiver viewed in the frequency domain. There was no restrictive stipulation in the analysis of Chapter III as to total system bandwidth--the results would apply, within reasonable tolerance, even to a degenerate system composed of a single-frequency sounding signal and an adjacent narrowband message signal. In such a case, the number of delay-line-model taps required to represent the channel filter would have diminished to a very few. The desired time-varying compensation of stored-reference symbols need not necessarily be done with a tapped delay line--other kinds of amplitude- and phase-adjustment circuits might work equally well.

The generality of the results of Chapter III with respect to absolute system bandwidth appears to disagree with a widely held belief that wideband signals are inherently necessary for efficiently combatting the undesirable effects of random multipath propagation. Part of the disagreement may be explained by considering the assumptions of the analysis, which tacitly imply a large alphabet of long-duration symbols. Long-duration symbols, in turn, provide a high degree of time diversity, so that the effects of narrowband selective fading are unimportant. If the use of short symbols is desired, a reasonable degree of frequency or space diversity or some form of segmented-symbol time diversity is important. One convenient way (but certainly not the only way) of achieving the desired diversity is the use of a wideband adaptive matched-filter implementation employing tapped delay lines.

If $T > 1/B_s$ and, either by choice or by necessity, detailed channel-sounding information is not available at the receiver, it is possible to combine the results of the several different coherent signal observations during T without first phase-correcting them. If the observation signal vectors are added vectorially, the resultant phase is random and carries little or no useful message information. A better procedure is to combine the vector magnitudes algebraically, ignoring phase angles, after applying weighting functions to get the best possible SNR in the resultant. The proper weighting function, in absence of a knowledge of the channel attenuation, is introduced by merely squaring the amplitudes of successive observations--the receiving procedure thus corresponds to making a simple measurement of total received-signal energy during T. Chapter IV has been devoted entirely to the detailed analysis of the performance of systems based on such a detection procedure.

For the case where the restriction $T < 1/B_s$ is satisfied, then the receiver decision can be made on the basis of a single, coherent measurement of the narrowband-symbol amplitude and phase during each interval T. A wide variety of modulation techniques can be used, and some can be implemented very simply. Where absolute channel gain and phase are not known, absolute significance cannot be attached to receiver observations of symbol phase or amplitude. Instead, individual observations may be compared in amplitude or phase with observations of one or more adjacent symbols, depending on how long a time interval the channel transfer function may be regarded as remaining constant. The adverse effects of the multipath channel on these techniques are frequently described in terms of intersymbol influence and selective-frequency fading. The intersymbol-influence problem can be avoided without undue loss of efficiency by ignoring overlapping portions of received symbols, provided T is appreciably greater than T_m. Thus, it is required that $1/B_s > T > T_m$, which is readily satisfied as long as $T_m B_s$ is much less than unity. Satisfying $T > T_m$ for a high information-transmission rate may require the use of a number of parallel frequency-division subchannels, but there is no intrinsic disadvantage in such a procedure. The selective-frequency-fading problem can be ameliorated with numerous schemes for frequency, space, polarization, and time diversity or with suitable error-correcting coding (which amounts to diversity).

While it is probable that future elaborations on simpler forms of narrowband multipath techniques will have neither the efficiency potential nor the adaptability to particular channel characteristics that are offered by techniques involving explicit channel-sounding procedures, it is possible that the difference in efficiency may be small, and hence, may be a secondary consideration compared to operating convenience and equipment simplicity. A laborious task of analyzing and comparing the performance of a wide variety of practical multipath techniques, under appropriately specified conditions, must be undertaken before any general conclusions can be drawn as to significant advantages of particular approaches.

B. PERFORMANCE OF SOME ELEMENTARY TECHNIQUES

As a rough indication of what may be accomplished with elementary techniques for random-multipath communication, consideration may be given to simple differential phase-shift systems having no provision for either frequency or space diversity. Let it be assumed that the condition $T_m < T < 1/B_s$ is easily satisfied, so that the effects of intersymbol influence are negligible and so that receiver decisions can be based on single, individual, coherent measurements of received-symbol phase.

In a differential phase-shift system, message information determines the incremental phase shift imposed on the transmitted signal between one symbol and the next. A change in the phase of the channel transfer function during transmission of an adjacent pair of symbols may cause a differential phase-measurement error to be introduced, but Juda and Skalbania [Ref. 17] have shown that this error must be sizable before it noticeably degrades the decision-error rate of a simple phase-shift system. Since the maximum frequency deviation introduced by the channel is $B_s/2$, it might appear that the mean differential-phase error in reception could never exceed $\pi B_s T$ radians. An observation of this kind might make some sense if the system were linear, but the very process of phase measurement implies a nonlinear receiver operation following linear filtering processes. The nature of channel phase errors is illustrated by the example of Fig. 12. These curves were measured over a transcontinental ionospheric multipath circuit at 15 Mc having $T_m \approx 3$ msec and $B_s \approx 1$ cps. A continuous sine wave was transmitted, and the amplitude and frequency of the received signal were simultaneously recorded. Relative phase was obtained from frequency recordings by integration.

A suitable differential phase-shift system, for a channel similar to that of Fig. 12, might employ symbols of duration many times T_m. If $T \approx 20$ msec is selected, it may be observed from a careful examination

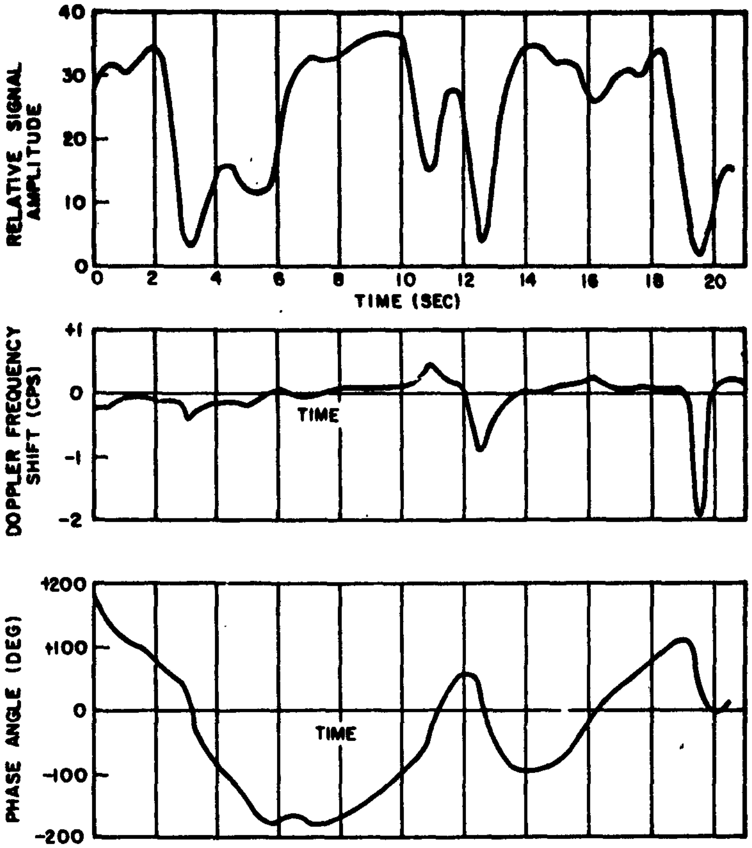


FIG. 12. TYPICAL TIME-VARYING AMPLITUDE- AND PHASE-RESPONSE FUNCTIONS OF A NARROWBAND MULTIPATH CHANNEL.

of Fig. 12 that the differential phase error occurring during the worst possible conditions is little more than 10 deg. Such an error occurs very rarely--only during short intervals of deep fadeout; yet even then it is of little significance. Even for values of T_mB_s greater than that specifically considered here, it appears that the decision-error rate may be determined accurately by merely averaging the error probability for a fixed channel, as a function of received signal strength, over the appropriate signal-fading probability distribution. Such a procedure may be carried out conveniently by expressing error probability and received-signal strength in terms of the power-requirement factor (energy per transmitted bit divided by n_o) that has been used in previous chapters. Thus,

$$\epsilon_{average} = \int_0^\infty \epsilon(\mathcal{F}_P)P(\mathcal{F}_P)d\mathcal{F}_P \tag{5.1}$$

Let the mean energy per received symbol be represented by $\mathcal{F}_{P_m}$ and let it be assumed that the rapid fading of the channel is characterized by Rayleigh statistics. By a change of variables from amplitude to amplitude squared, it is readily shown that the probability that a pair of adjacent symbols, selected at random, have energies corresponding closely to $\mathcal{F}_P$ is given by

$$P(\mathcal{F}_P) = \frac{1}{\mathcal{F}_{P_m}} \exp[-(\mathcal{F}_P/\mathcal{F}_{P_m})] \tag{5.2}$$

The subject of decision-error rates for digital receivers has been dealt with extensively in the communication-system literature. An ideal, binary, polarity-modulated system (assuming a perfect phase reference, which is never precisely available for a random channel), operating in the presence of additive gaussian noise, has an error rate given by [Ref. 18]

$$\epsilon(\mathcal{F}_P) = \int_{-\infty}^{-\sqrt{2\mathcal{F}_P}} \frac{1}{\sqrt{2\pi}} \exp(-x^2/2)dx \tag{5.3}$$

Such an error characteristic, which is illustrated in Fig. 13, can be approximated very accurately by the simple exponential expression of Eq. (5.4), with $C = 1.242$.

$$\epsilon(\mathcal{F}_P) = \frac{1}{2} \exp(-C\mathcal{F}_P) \tag{5.4}$$

A similar characteristic for a binary π differential-phase-shift system, as derived by Lawton [Ref. 19], is given precisely by Eq. (5.4) with $C = 1.0$. The error rate of a dual-binary, $\pi/2$ differential-phase-shift system, described by Doelz, Heald, and Martin [Ref. 20], is similarly described by Eq. (5.4), with $C = 0.551$.

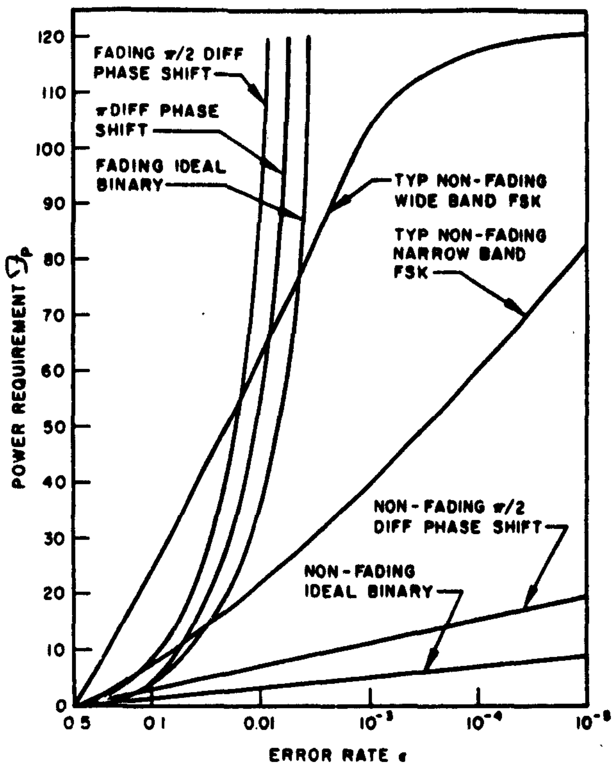


FIG. 13. RELATIONS BETWEEN SYMBOL ENERGY AND ERROR RATE.

The integration indicated in Eq. (5.1) may be carried out readily for functions having the general form of Eqs. (5.2) and (5.4). The result is

$$\epsilon_{\substack{\text{average}\\ \text{(Rayleigh}\\ \text{fading)}}} = \frac{1}{2(C\mathcal{S}_{p_m} + 1)} \tag{5.5}$$

Several error-rate curves for fading channels, obtained in the manner described, have been included in Figs. 13 and 14. The typical curves for FSK systems are taken from Doelz, Martin, and Heald [Ref. 20].

The effects of error equivocation must be introduced to obtain relations between power and maximum rate of transfer of information through the system. Equation (2.8) expresses the proper correction for a symmetrical digital system with independent occurrence of errors. By applying such a correction to Fig. 13, one obtains the results shown in Fig. 14.

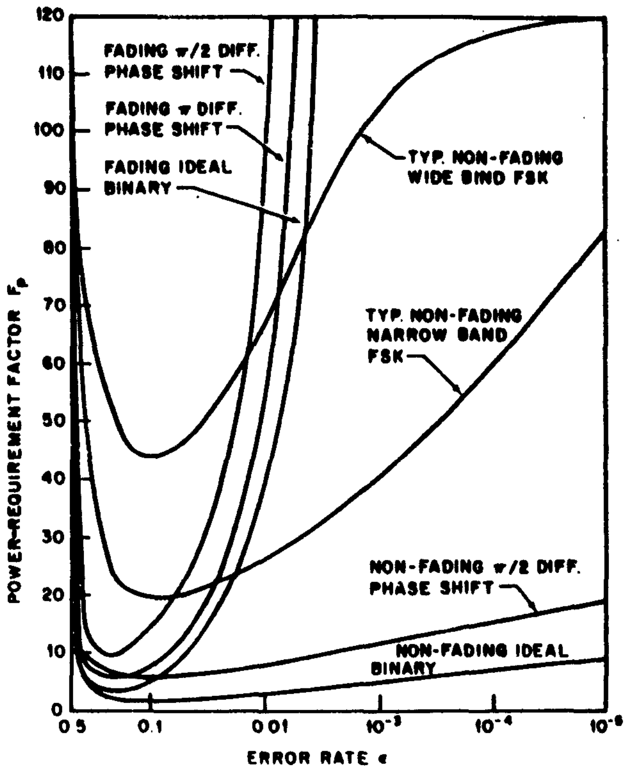


FIG. 14. POWER-REQUIREMENT FACTOR VS ERROR RATE.

The assumption of independent random error occurrence that has been used in obtaining the curves for the fading cases of Fig. 14 yields results that are pessimistic in some respects. The effect of errors, given by the second term of Eq. (2.8), may be considerably less than that calculated, for two reasons. In the first place, a degree of error dependence is introduced by the differential modulation technique and by the common level of fading experienced during adjacent symbol intervals; such dependence yields a conditional digital-channel entropy less than the value that has been calculated. Considerable progress has been made, for example, in devising systematic error-correcting codes to take advantage of error clustering [Refs. 21, 22]. In the second place, the receiver may derive a symbol-by-symbol time-varying error-probability estimate from observations of the amplitude of received signals and noise.

It is clear from Fig. 14 that an individual narrowband fading channel must be operated at a relatively high error rate in order to optimize power requirements. The penalty for failure to operate with a high average receiver-decision error rate is generally much more severe than was the case with the incoherent-detection techniques considered in Chapter IV. While presently available error-coding techniques for reducing high receiver-decision error rates to the much lower values usually desired in practice may be rather inefficient, greater application of advances in digital-data handling and storage capabilities can be expected to improve the situation, especially if improved methods are developed for exploiting error dependence and error knowledge of the kind described.

It should be noted that many of the common forms of diversity reception used for digital communication over fading channels are by nature elementary, highly redundant methods of parallel error-correction coding. For a given total effective antenna aperture, they offer little special advantage over purely serial coding except in the elimination of information storage requirements in coding and detecting. The mathematical treatment of parallel error-correction coding generally may be handled by merely reinterpreting the results of serial coding procedures. For this reason, it is unnecessary to give special consideration to parallel-diversity techniques when evaluating performance potentials.

C. COMPARISON OF TECHNIQUES

In the analysis of techniques so far, emphasis has been placed on the conservation of signal power required to accomplish a given communication task. To a certain extent, the power-requirement curves that have been presented enable one to draw conclusions about the relative advantages of different techniques. In many applications, however, a need for efficient use of available frequency allocations may force one to compromise power efficiency. A graphical presentation of the relationship between the required power per unit information-transfer rate and the required bandwidth allocation, for specific systems and specific types of channels, is a useful aid when such a compromise is to be made.

It is possible to obtain a limiting relation, between required power and required bandwidth, that cannot be improved upon under any circumstances. With regard to the channel model of Fig. 2, it is clear that the rate of information transfer from x to y can never exceed the rate of transfer of information from u to y. Shannon [Ref. 4] has shown that, for a signal $u(t)$ limited to power S and bandwidth W and corrupted by additive, independent, white, gaussian noise of power-spectral density n_o, the rate of information transmission is always limited by

$$R \leqq W \log_2\left(1 + \frac{S}{n_o W}\right) = W \log_2(SNR + 1) \tag{5.6}$$

Thus, one may obtain

$$F_P \triangleq \frac{S}{n_o R} = \left(\frac{S}{n_o W}\right)\left(\frac{W}{R}\right) \geqq \frac{SNR}{\log_2(SNR + 1)} \tag{5.7}$$

and

$$F_W \triangleq \frac{W}{R} = \frac{F_P}{SNR} \geqq \frac{1}{\log_2(SNR + 1)} \tag{5.8}$$

The SNR parameter may be eliminated from the inequality of Eq. (5.8) to obtain the universal limit

$$F_P \geqq \left(\frac{W}{R}\right)(2^{R/W} - 1) \tag{5.9}$$

which has been plotted in Fig. 15.

It is also possible to obtain relations between F_P and F_W for specific examples of each type of system that has been studied. A comparison of such curves with one another and with the ultimate performance limit of Eq. (5.9) is more revealing than merely making power-requirement comparisons without regard to bandwidth occupancy.

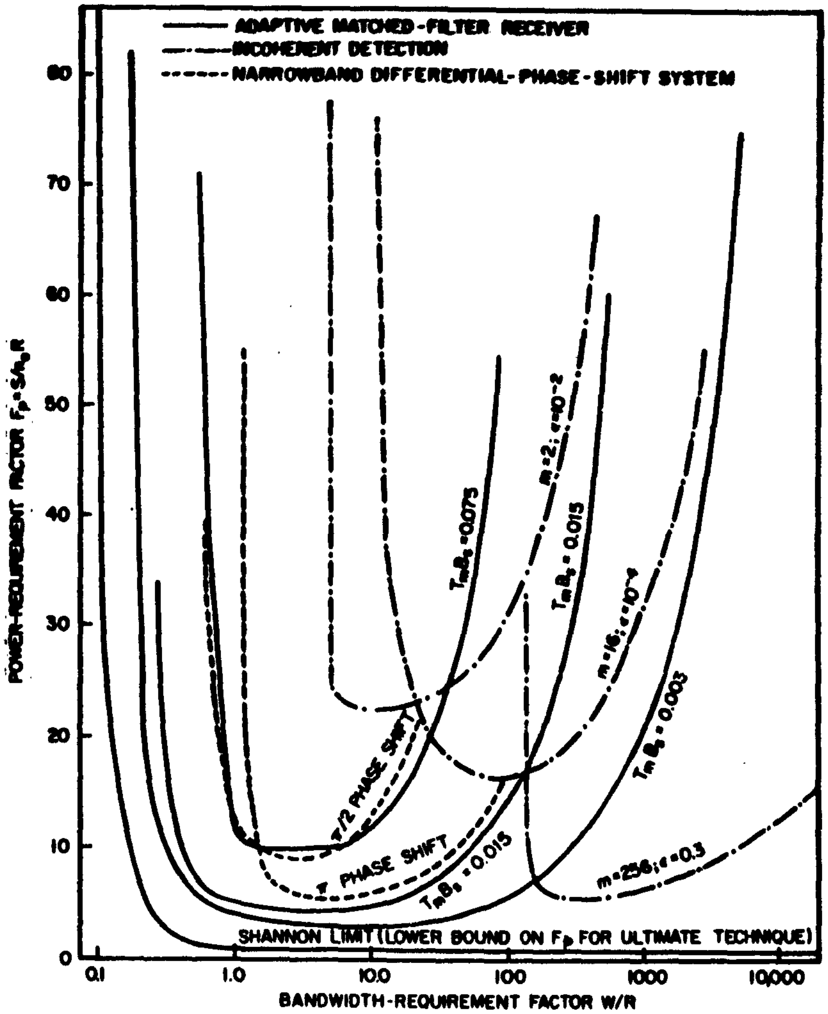


FIG. 15. POWER REQUIREMENTS VS BANDWIDTH REQUIREMENTS.

In the case of the adaptive matched filter, Eq. (3.23) gave bounds on F_p as a function of channel parameters, signal-to-noise ratio, and internal system-design parameter k. Corresponding values of F_W are readily obtained from the relation $W/R = F_p/SNR$. Curves representing the upper bound on F_p have been shown in Fig. 15 for selected values of the product T_mB_s. The individual curve coordinates have been calculated by treating SNR as the independent system variable and by choosing an optimum value of k for each operating condition.

In the case of systems based on incoherent-detection techniques, it is necessary to account for the transmitted-pulse duty factor before W/R may be determined. If the system is designed to use all quiescent time between pulses in such a way as to obtain maximum alphabet size for given T and given W, then

$$W \approx m\left(B_s + \frac{1}{T}\right)\frac{T}{T_S} \tag{5.10}$$

and

$$F_W \triangleq \frac{W}{R} = \left[\frac{m\left(B_s + \frac{1}{T}\right) T}{T_S}\right]\left(\frac{n_o F_P}{S}\right) = \frac{mF_P}{\widehat{SNR}} \tag{5.11}$$

Where quiescent time is not fully used, F_W is accordingly greater. In the pertinent curves of Fig. 15, it has been assumed that maximum use has been made of available time. $\widehat{SNR}$ has been treated as an independent system variable in obtaining particular curves.

In general, the required bandwidth of a differential-phase-shift system is directly proportional to the transmitted bit rate $\mathfrak{R}$. If SNR is varied independently at constant $\mathfrak{R}$, the error rate is affected, and thus R varies as indicated by Eq. (2.8). The resulting variations in F_W and F_P have been plotted in Fig. 15 for several cases, where the same assumption of error independence used in Chapter V-B has been introduced.

While Fig. 15 indicates that the adaptive matched-filter system has a potential advantage over other techniques considered, for reasonably small values of $T_m B_s$, the magnitude of the advantage does not appear to be decisive. For many types of multipath channels, it seems that limited additional advantage can be gained by providing greater freedom to optimize the adaptive matched-filter receiver; even a generally optimum system could save no more than a few decibels of power.

In the case of channels with large values of $T_m B_s$, it appears that the incoherent-detection techniques can achieve very good power economy, but the bandwidth required to do so will be far greater than is normally provided to accomplish a given communication task on other types of channels.

VI. CONCLUSIONS

A. SIGNIFICANCE OF RESULTS

A primary endeavor in this report has been to find useful ways of making performance evaluations for specific types of communication systems. The results that have been obtained complement the work of previous researchers who have concerned themselves primarily with synthesizing general forms of systems without specifically undertaking detailed optimization or performance analysis for practical random-multipath channels.

In Chapter III, bounds have been obtained for the rate of communication possible when an adaptive matched-filter reception technique is used. Previous analyses of comparable techniques and previous investigations of information communication rates for random multiplicative types of channels have been successful, to the writer's knowledge, only for trivial channel assumptions that are rarely, if ever, met in practice. In contrast, the necessary channel assumptions that have been made in Chapter III appear to be reasonably well satisfied in connection with a wide variety of multipath-propagation circuits having considerable practical importance.

Several of the incidental results, obtained in connection with the analyses of Chapter III, appear to be of some significance in themselves. It is possible that wide application can be found for the general relations, derived in Appendix B, for the effect of independent channel-sounding information on the message-information communication rate. The bounds obtained for the self-entropy of a signal received over a fading multipath channel may also be independently of interest. Other matters that may fall into the special-interest category are the limitations of correlation-detection procedures when reference symbols are uncertain.

A comprehensive analysis of the performance of a broad class of receiving techniques based on energy measurement has been undertaken in Chapter IV. The results are of particular interest in situations where variations in the channel transfer-function are too rapid for it to be satisfactorily measured, but application is possible and perhaps desirable on more favorably behaved channels, as well. The results of Chapter IV are, incidentally, found to be generally applicable to a wide variety of post-detection diversity-combining techniques for fading channels, including many combinations of frequency, time, space, and polarization diversity.

In Chapter V, a limited analysis has been made of some elementary multipath-communication techniques, and a method of comparing the performance of different types of techniques has been suggested. An absolute performance bound applicable to all pertinent types of systems has been obtained. For many common types of multipath circuits, it appears that very little improvement is possible over the particular adaptive matched-filter techniques considered in Chapter III. Furthermore, the performance potential of elementary narrowband techniques is not found to be decisively unfavorable compared to that of adaptive matched-filter receiving techniques. When one considers that simple FSK and AM communications systems are still in wide use on nonfading channels in spite of appreciable improvements that have been made (for example, see Fig. 13), it becomes questionable, in many applications, whether matters of convenience and operating simplicity may not continue to favor the use of elementary, appropriately designed narrowband systems instead of the more sophisticated techniques that could be used. A much more significant opportunity for highly sophisticated systems to excel arises in connection with channels that behave more poorly than most ionospheric multipath circuits.

B. SUGGESTIONS FOR FURTHER INVESTIGATION

There are almost unlimited possibilities for extending and improving the analytical treatment of multipath-communication problems, but finding those that are both mathematically tractable and significantly profitable may be very difficult. Some of the assumptions that have been made concerning the channel raise interesting possibilities. Nonwhite and/or nonstationary additive noise could probably be handled by determining the noise characteristics as a function of time and by prior-processing the received signal in such a manner that additive noise appears to be stationary-white to the receiver. As far as the signal is concerned, the added perturbations can be regarded as having arisen in the channel. Except for the uncertainty in measuring noise characteristics, the analysis problem might not be affected very much by the suggested modification.

The possibility of nongaussian additive noise, especially that arising from electrostatic discharge at ionospheric-reflection frequencies, is worthy of consideration. It is quite possible that the effects of special noise statistics might not be much different on random multipath channels than on channels where random disturbances are exclusively additive. In such an event, impulse noise might be handled analytically merely by substituting a known equivalent gaussian noise (somewhat analogous to determining the entropy power of nongaussian noise), and physically by including some special signal-processing circuits in the receiver.

Various filters, in the systems that have been considered, could be optimized for specific doppler-spreading characteristics that arise in practice, but the consequences are likely to be relatively minor. It was suggested, during consideration of adaptive matched-filter techniques, that detailed consideration might be given to the statistics of tap multipliers in the delay-line representation of particular types of channels. A consequent time-varying optimization of auxiliary weighting functions is likely to yield greatest improvement in transfer-function measurements in the case of ionospheric multipath channels, which tend to have a limited number of discrete propagation modes. The improvement theoretically possible in overall performance may be too limited to justify much expenditure of analytical or experimental measurement effort in the case of ionospheric channels, however. Greater benefit is theoretically possible on poorly behaved scatter-multipath circuits, but here the impulse response of the channel is likely to be more homogeneous, and thus, more accurately represented by the assumptions made in Chapter III.

Precise consideration of the effects of signalling alphabets that are of practical size and duration would be an enlightening extension of the results of Chapter III, but the analytical problems may be very difficult. An experimental investigation of such effects might be accomplished satisfactorily by simulating an adaptive matched-filter system on a digital computer. Experimental verification of analytical results by simulation would be particularly attractive because of the ease with which system-design parameters and channel-description parameters could be varied.

The availability of symbol-by-symbol estimates of nonstationary error probability and the occurrence of specific error dependences in certain fading narrowband digital communications systems suggest interesting new possibilities for error-correcting coding investigations. Explicit performance evaluations for improved types of random multipath systems based on narrowband realization of adaptive matched-filter techniques would also be interesting, and would probably yield results of immediate practical value.

If one considers large, slow variations in such general channel parameters as mean signal attenuation, frequency dispersion, differential propagation-time dispersion, and additive noise level, a need arises for adapting the transmitted signal to the channel. Conveying the necessary channel information back to the transmitter for such a purpose requires a limited feedback capability, but is unlikely to influence the fundamental design considerations very much. The possible use of feedback to permit adapting transmissions to rapid variations in the specific transfer function of the channel would apparently require drastic revision in analytical techniques. A general investigation of such types of systems would be interesting and might lead to useful results for some types of practical multipath channels.

APPENDIX A. INTERSYMBOL-INFLUENCE CONSIDERATIONS

If a communication system sends single symbols of duration T from an alphabet of total size m, the rate of communication of information can never exceed

$$R \leq \frac{1}{T} \log_2(m) \tag{A.1}$$

To avoid excessive system complexity, it may be desirable to restrict m to a reasonably small size. In such a case, R is normally increased by shortening T. As T becomes of comparable duration to, or shorter than, T_m on a multipath channel, there may be added complications in the implementation of an adaptive matched-filter communication system.

One difficulty arises because multipath components of one symbol, arriving by paths with longer delays, may be totally indistinguishable from components of a subsequent symbol arriving via shorter paths. Thus, there may arise a form of coherent intersymbol influence that is in no way accounted for in the analysis of Chapter III. This problem can be avoided by providing n different m-member sets of symbols (such that $nT > T_m$) and changing sets every symbol. While the total number of symbols has increased to nm, this is fewer than the $(m)^n$ symbols that would be required to achieve the same R if symbol duration were simply increased by factor n. In addition, since only m of the symbols need be made available for possible transmission at any particular time, it is possible to generate all nm symbols with only m periodic waveform generators, each with period T_m or greater. In effect, such a procedure was used in the Rake system [Ref. 8].

A second difficulty arises because, in the process of modifying reference symbols, they may be stretched in time by appreciable factors. If a continuous succession of received signals is to be detected, then a total of $2m$ or more reference-symbol-compensation filters may have to be synthesized to avoid a form of inter-reference-symbol influence. Another way of avoiding this problem involves a major manipulation of the form of the adaptive matched-filter receiver. With regard to the channel model of Fig. 2, the desired cross-correlation product between a received symbol and its reference is given by

$$I^{(j)} = \sum_{i=1}^{n} \int_{0}^{T+T_m} [u(t) + n(t)][x^{(j)}(t - i\tau)r_i(t)a_i(t)]dt \qquad (A.2)$$

The function $u(t)$ may be written

$$u(t) = \sum_{i}^{n} x(t - i\tau)r_i(t) \qquad (A.3)$$

By interchanging the order of summation and integration, one obtains

$$I^{(j)} = \int_{0}^{T+T_m} \sum_{i=1}^{n} \left[\sum_{k=1}^{n} x(t-k\tau)r_k(t) + n(t)\right] \left[x(t-i\tau)r_i(t)a_i(t)\right] dt \qquad (A.4)$$

In Eq. (A.4), $x(t - i\tau) = 0$ except for $0 \leq t - i\tau \leq T$. If the substitution $t = t' + i\tau$ is made, one obtains

$$I^{(j)} = \int_{0}^{T} \sum_{i=1}^{n} \left[\sum_{k=1}^{n} x(t' - k\tau + i\tau)r_k(t' + k\tau) + n(t' + i\tau)\right] \left[x(t')r_i(t' + i\tau)a_i(t' + i\tau)\right] dt' \qquad (A.5)$$

For the condition $T_m B_s < 1$ applicable to the operation of the system, and for T short compared to T_m, the $r_i(t)$ are essentially constant in the interval $0, T$. Thus $r_i(t' + i\tau) \approx r_i(t')$, and

$$\sum_{k=1}^{n} [x(t' - k\tau + i\tau)r_k(t' + i\tau)] + n(t' + i\tau) \cong u(t' + i\tau) + n(t' + i\tau) = y(t' + i\tau) \qquad (A.6)$$

Therefore

$$I^{(j)} = \int_0^T \sum_{i=1}^{n} [y(t' + i\tau)][x(t')\bar{r}_i(t')a_i(t' + i\tau)]dt' \quad (A.7)$$

Again interchanging the order of summation and integration, one obtains

$$I^{(j)} = \sum_{i=1}^{n} \int_0^T [y(t' + i\tau)][x(t')r_i(t')a_i(t' + i\tau)]dt' \quad (A.8)$$

The desired cross-correlation product, rearranged as indicated in Eq. (A.8), may readily be formed by processing the received signal with a tapped delay line, separately cross-correlating the unmodified stored reference symbol with the output from each tap over a time interval T (with appropriate multipliers and auxiliary weighting functions introduced), and summing the results. With this arrangement, no explicit filtering of stored reference symbols is required and no inter-reference-symbol interference can arise.

The discussion and the manipulation of cross-correlation products that have been presented should help to clarify some questionable points raised in the explanation of the Rake system. Price and Green [Ref. 8] have indicated that the passing of the received signal (rather than the stored references) through the delay line was fundamentally responsible for the elimination of coherent intersymbol-influence effects in the case of short symbols. Such a conclusion is correct, but only if one is speaking of intersymbol influences originating in a faulty receiver. There are clearly other ways of eliminating such effects that do not involve passing the received signal through a delay line. On the other hand, authentic coherent intersymbol influences arising in the multipath channel cannot be eliminated by manipulating the form of the receiver. They can be eliminated, however, by properly choosing unlike sets of symbols for adjacent use in time, a procedure which was also done in the Rake system.

Important practical advantages may arise from use of the modified system (even where $T > T_m$), particularly where m is large. The revised form of system implementation requires a greater number of multipliers, but no more than one tapped delay line is ever needed, no matter how large the alphabet becomes.

There is an alternate way of increasing R without providing either unduly large m or unduly short T. A primary difficulty is that, for constant T, m usually must increase exponentially with R, since the amount of information per symbol is limited to log(m). The limitation of Eq. (A.1) applies only where a single symbol is transmitted at a time, however. If n independent symbols are transmitted concurrently, then the limitation would be

$$R \leq \frac{n}{T} \log_2(m) \tag{A.9}$$

The total number of symbols is again nm, and for constant m, R may increase linearly with the total number of symbols. The n subsets of symbols may overlap in frequency, or they may be selected from orthogonal-frequency sets. In the latter case, it may be desirable to provide better frequency diversification by interlacing the frequency-spectral components of the subsets. Such a procedure does not appear to cause appreciable difficulties as long as $T_m B_s$ is very small. The actual implementation of the described system might well take the form of a set of n narrowband subchannels, each with alphabet size m and each with a satisfactory degree of coherent frequency-diversity combining. The signals transmitted through the different subchannels could be completely independent, if desired, and the use of a tapped delay line might be entirely unnecessary if alternate implementation techniques described in Sec. A, Chapter V were employed.

It is evident that adaptive matched-filter receiving techniques, such as those described in Chapter III, may be applied directly to the transmission of analog information, provided the correlation time of the analog signals is long compared to T_m. For example, the analog signals might simply amplitude-modulate one or more long, periodic symbols. Such a procedure might conceivably be applied to analog signals with short correlation times, but the spectra of message-bearing signals would then be spread to interfere with the proposed channel-sounding signals.

APPENDIX B. EFFECTS OF CHANNEL-SOUNDING INFORMATION ON COMMUNICATION RATE

Suppose that a receiver continuously obtains and uses information about the transfer function of a channel. Let the channel information be represented by an arbitrary number of unspecified time functions, $z_1(t)$, $z_2(t)$, ... $z_n(t)$. The set $\{z_i(t)\}$ could be measured delay-line channel-model tap multipliers, or they might be analogous gain and phase functions of time measured at uniform intervals in the band of frequencies occupied by the receiver signals. The $\{z_i(t)\}$ might alternately be any arbitrary functions derived from the channel transfer characteristic; for example, they might be a modified set of stored reference symbols.

If the $\{z_i(t)\}$ are statistically independent of which particular message symbol or combination of symbols has been transmitted, and if the channel information contained in the $\{z_i(t)\}$ is used by the receiver in an optimum manner, the effect on the achieved message-information communication rate may be precisely determined. Consider the signal designations indicated in Fig. 16, and let the statistics of the various signals be represented by finite-dimensional probability-distribution functions, as was suggested in Chapter II.

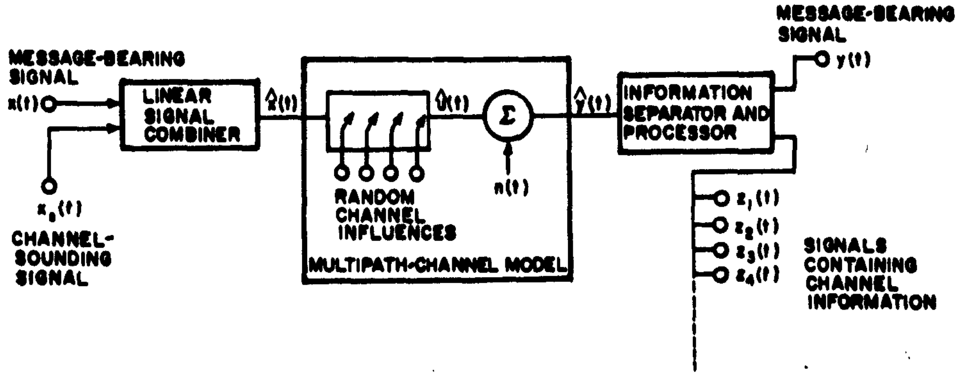


FIG. 16. BLOCK DIAGRAM OF A POSSIBLE COMMUNICATION SYSTEM.

For the purpose of determining the rate of communication of information through the system, $x(t)$ may be regarded as the message input, and $y(t)$ and all of the $\{z_i(t)\}$ may be regarded compositely as the received signal. Although the $\{z_i(t)\}$ are independent of the message, they contain appreciable amounts of message information from the receiver's point of view. The situation is analogous to that of an erring binary communication system with a "safety observer" to report errors. The occurrence of errors, and consequently the reports of the observer, may be completely independent of the sequence of input digits that is transmitted, yet the rate of communication achieved will be directly improved if the receiver is allowed access to error reports.

From Shannon [Ref. 4], one obtains

$$R = H(x) - H_{y,\{z\}}(x) = H(y,\{z\}) - H_x(y,\{z\}) \tag{B.1}$$

By expansion,

$$R = H_{\{z\}}(y) + H(\{z\}) - H_x(y) - H_{x,y}(\{z\}) \tag{B.2}$$

But

$$H(x,y,\{z\}) \equiv H(x) + H_x(y) + H_{x,y}(\{z\}) \equiv H(\{z\}) + H_{\{z\}}(x) + H_{x,\{z\}}(y) \tag{B.3}$$

from which

$$H(\{z\}) - H_x(y) - H_{x,y}(\{z\}) = H(x) - H_{\{z\}}(x) - H_{x,\{z\}}(y) \tag{B.4}$$

Substituting Eq. (B.4) into Eq. (B.2), one obtains

$$R = H_{\{z\}}(y) + [H(x) - H_{\{z\}}(x) - H_{x,\{z\}}(y)] \tag{B.5}$$

Since the $\{z_i(t)\}$ are independent of x, $H_{\{z\}}(x) = H(x)$. Thus,

$$R = H_{\{z\}}(y) - H_{x,\{z\}}(y) \tag{B.6}$$

If the $\left\{z_i(t)\right\}$ used in the receiver decision process are the set $\left\{u_{est}^{(j)}(t)\right\}$, as is the case for the system analyzed in Chapter III, then clearly

$$R = H_{\left\{u_{est}^{(j)}\right\}}(y) - H_{x,\left\{u_{est}^{(j)}\right\}}(y) \tag{B.7}$$

which is the result introduced into the analysis at Eq. (3.2).

APPENDIX C. BOUND ON THE ENTROPY OF A RECEIVED SIGNAL

Shannon [Ref. 4] has shown that if an ensemble x having an entropy $H(x)$ per degree of freedom in bandwidth W' is passed through a known, fixed, linear filter with a transfer characteristic $Y(f)$, the output ensemble has an entropy (in bits per degree of freedom) given by

$$H(u) = H(x) + \frac{1}{W'}\int_{W'} \log_2[|Y(f)|]^2 df \qquad (C.1)$$

Equation (C.1) was obtained by regarding the filter as a diagonalized coordinate transformation matrix operating on the frequency components of x.

In the case of a random multipath channel, one is concerned with a time-varying linear filter that is not precisely known. For an uncertain fixed-filter characteristic, one may write

$$H(u) \geq H_Y(u) = H(x) + \frac{1}{W}\int_{W} \log_2[|Y(f)|]^2 df \qquad (C.2)$$

Equations (C.1) and (C.2) may be expected to apply to a time-varying situation as long as the filter is varying so slowly, relative to the signal bandwidth, that a quasi-stationary analysis is allowable. (The usual Fourier-transform relations are assumed to hold between correlation functions and power spectra on a finite-observation-interval basis in the derivation of the adaptive matched-filter multipath system [Ref. 8].)

For the random multipath-channel assumptions made in Chapter III, Y is Rayleigh distributed in time and frequency with a mean of unity. Thus,

$$p(|Y|) = 2|Y|e^{-|Y^2|} \qquad (C.3)$$

Since it has been assumed that the statistical processes of the channel are ergodic, the .esired geometric-mean filter gain may be obtained from an ensemble average. Thus,

$$H(u) \geq H(x) + \int_Y p(|Y|)\log_2(|Y|^2)d|Y| \triangleq H(x) + K \qquad (C.4)$$

where

$$K \triangleq \int_0^\infty 2|Y|e^{-|Y^2|} \log_2(|Y^2|)d|Y| \qquad (C.5)$$

If the substitutions $z = |Y^2|$ and $\log_2(x) = \log_e(x)\log_2(e)$ are introduced, then $dz = 2(|Y|)dY$, and

$$K = \log_2(e)\int_0^\infty e^{-z} \log_e(z)dz \triangleq A \log_2(e) \qquad (C.6)$$

From tables of definite integrals [Ref. 23, Table No. 256], $A = -0.5772$. Therefore, $K = -0.833$ and

$$H(u) \geqq H(x) - 0.833 \qquad (C.7)$$

In calculating communication-system signal entropies, it is proper to assume that the set of possible transmitted symbols can be known <u>a priori</u> to the receiver. If these symbols are operated on in accordance with receiver estimates of $Y(f)$ to obtain a set of possible $u_{est}^{(j)}(t)$, knowledge of the $\left\{u_{est}^{(j)}(t)\right\}$ can imply no more information about $u(t)$ than knowledge of $Y(f)$ and the set of possible transmitted symbols. Thus,

$$H_{\left\{u_{est}^{(j)}\right\}}(u) \geqq H(x) - 0.833 \qquad (C.8)$$

For an input signal x of variance S' that is gaussian and of uniform power-spectral density in W', one has

$$H_{\left\{u_{est}^{(j)}\right\}}(u) \geqq 2W'\left\{\log_2 \sqrt{2\pi e S'} - 0.833\right\} \text{ bits/sec} \qquad (C.9)$$

or

$$H_{\left\{u_{est}^{(j)}\right\}}(u) \geqq W' \log_2[2\pi e(0.316S')] \qquad (C.10)$$

Thus,

$$\bar{S}(u)\Big|_{\left\{u_{est}^{(j)}\right\}} \gtrsim 0.316S' \qquad (C.11)$$

a result that has been used in Eq. (3.4).

APPENDIX D. DETERMINATION OF AUXILIARY WEIGHTING FUNCTIONS

To obtain auxiliary weighting functions $a_i(t)$ for modifying measured values of tap multipliers in the adaptive matched-filter receiver, the following assumptions have been made:

1. The noises arising in measurement of the individual $r_i(t)$ are independent, gaussian, zero-mean, and equal in variance.
2. The $r_i(t)$ are independent, gaussian, zero-mean, and equal in variance.
3. The weighting functions are not allowed to vary with time.

Consider a general system model applicable, irrespective of the above restrictions. Since received-signal contributions arriving via different taps of the channel delay-line model of Fig. 2 have previously been assumed to be uncorrelated, the $a_i(t)$ may be chosen to minimize the individual variances of each of the i error functions included in Eq. (3.7). The manner in which the $e_i(t)$ arise may be represented by the block diagram of Fig. 17.

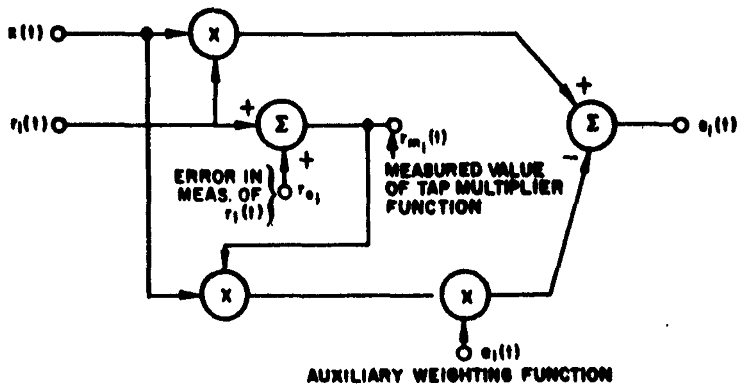


FIG. 17. SIGNAL-GENERATION MODEL USED IN OBTAINING WEIGHTING FUNCTIONS.

The function $e_i(t)$ may be described analytically as indicated in Eq. (D.1).

$$
\begin{aligned}
e_i(t) &= x(t)r_i(t) - x(t)r_{m_i}(t)a_i(t) \\
&= x(t)r_i(t)[1 - a_i(t)] - a_i(t)x(t)r_{e_i}(t)
\end{aligned}
\tag{D.1}
$$

In general, a_i can be a function of $r_{m_i}(t)$ and of the <u>a priori</u>-known joint-probability statistics of the $r_i(t)$, but not of the $r_i(t)$ functions themselves, since these are not available to the receiver. Thus, the variance of $e_i(t)$ is expressed by

$$\sigma_{e_i}^2 = \sigma_x^2\left\{\int_{r_i} P(r_i) r_i \left[1 - a_i\left(\left\{r_i + n_i\right\}\right)\right]^2 dr_i + \sigma_{r_{e_i}}^2 \int_{r_i} P(r_i)\left[a_i\left(\left\{r_i + n_i\right\}\right)\right]^2 dr_i\right\} \tag{D.2}$$

In a general optimization, a_i might be some particular function of the set of $r_{m_i}(t)$, and thus a_i would be an implicit function of time. Determination of the optimum function $a_i(r_{m_1}, r_{m_2}, \ldots r_{m_n})$ would depend on the particular joint-probability description applicable to the r_i. However, for the particular assumptions that have been made, a_i cannot vary with time and thus can be only a constant. Introducing such a constraint into Eq. (D.2), one obtains

$$\sigma_{e_i}^2 = \sigma_x^2\sigma_{r_i}^2(1 - a_i)^2 + \sigma_x^2\sigma_{r_{e_i}}^2 a_i^2 \tag{D.3}$$

Taking the derivative of Eq. (D.3) with respect to a_i and setting it equal to zero yields

$$a_i = \frac{\sigma_{r_i}^2}{\sigma_{r_{e_i}}^2 + \sigma_{r_i}^2} = \frac{\sigma_r^2/\sigma_{r_e}^2}{1 + \sigma_r^2/\sigma_{r_e}^2} \triangleq \frac{\gamma}{1 + \gamma} \tag{D.4}$$

as the optimum weighting function for the specified conditions. Substituting Eq. (D.4) into Eq. (D.2), one obtains

$$\sigma_{e_i}^2 = \frac{\sigma_x^2\sigma_r^2}{1 + \gamma} \tag{D.5}$$

From the relation $\sigma_r^2 = 1/n$ given in Eq. (3.8), one obtains

$$\sigma^2_{u_{error}} = n\sigma^2_{e_1} = \frac{\sigma_x^2}{1+\gamma} = \frac{kS}{1+\gamma} \tag{D.6}$$

APPENDIX E. AN OPTIMUM RECEIVER EMPLOYING NOISY REFERENCE SYMBOLS

In the communication channel of Fig. 18, transmitted signals of average power S and bandwidth W are drawn from a white-gaussian noise source. During reception, independent white, gaussian noise of power-spectral density n_o is added to the transmitted signal. It is assumed that the receiver cannot know the allowed set of possible transmitted symbols precisely a priori. A set of uncertain reference symbols, corrupted by individual, independent, random, additive, white noise signals of variance N_R, are available to the receiver, as indicated.

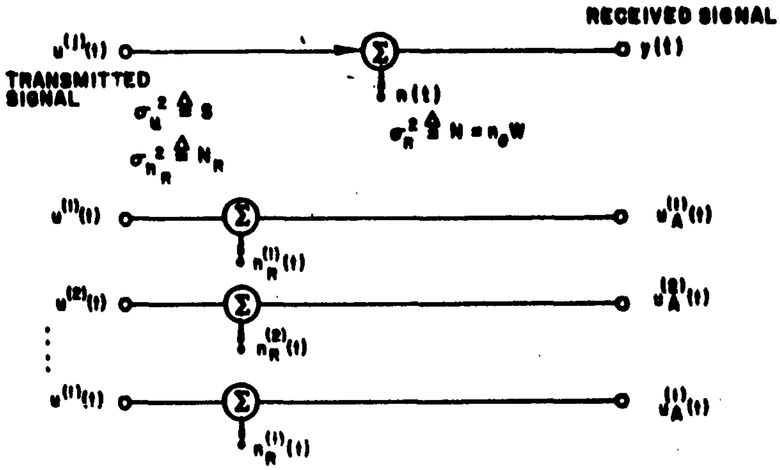


FIG. 18. COMMUNICATION SYSTEM CONSTRAINED TO USE NOISY REFERENCE SYMBOLS.

From the receiver's point of view, the probability of a correct decision is merely the probability that $y(t)$ is most similar, according to some appropriate criterion, to $u_A^{(j)}(t)$. It makes no difference to the receiver how the signals arise--the probability of error would be the same if $u_{est}^{(j)}(t)$ were the transmitted signal, as long as $y(t)$ were not affected. It is impossible to determine, from terminal observation of $u^{(i)}(t)$ and $u_A^{(i)}(t)$ which one is cause and which one is effect. It can only be known that they are both white, bandlimited, gaussian signals and that their covariance is equal to S.

If $u_A^{(j)}(t)$ is regarded as the transmitted signal, then the channel of Fig. 18 may be reinterpreted as shown in Fig. 19.

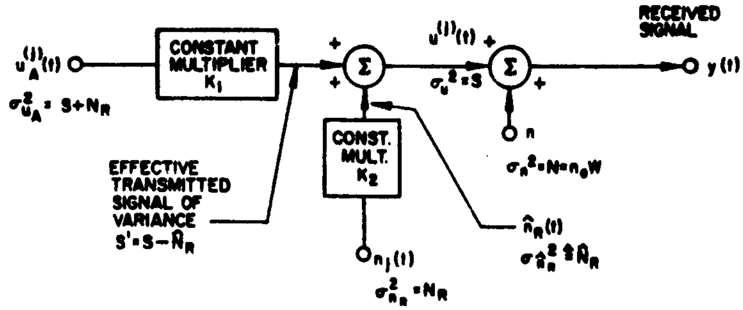


FIG. 19. OPERATIONALLY EQUIVALENT COMMUNICATION SYSTEM THAT IS FREE TO USE PERFECT REFERENCE SYMBOLS.

The receiver may now be regarded as having a perfect set of reference symbols, but the (fictitious) transmitted signal now undergoes additional perturbation in passing through the channel. In order to preserve the former variances of $u_A(t)$ and $u(t)$ and their former cross-correlation coefficient, it may readily be determined that

$$K_1^2 = \frac{S - \hat{N}_R}{S + N_R} = \left(\frac{S}{S + N_R}\right)^2 \;;\quad K_2^2 = \frac{S}{S + N_R} \;;\quad \text{and} \quad \hat{N}_R = \frac{SN_R}{S + N_R}$$

The resulting system may be precisely represented by the simplified diagram of Fig. 20.

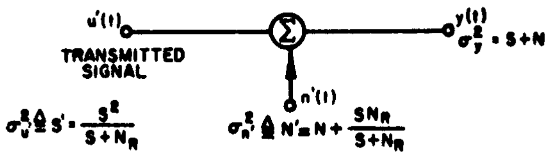


FIG. 20. SIMPLIFIED DIAGRAM OF COMMUNICATION-CHANNEL MODEL.

Since the signals are all still white, gaussian, and band-limited to W, the maximum rate of communication may be written by inspection.

From Shannon [Ref. 5],

$$R = W \log_2 \left(\frac{S' + N'}{N'} \right) = W \log_2 \left\{ \frac{\frac{S^2}{S + N_R} + N + \frac{SN_R}{S + N_R}}{N + \frac{SN_R}{S + N_R}} \right\} \tag{E.1}$$

or simplifying,

$$R = W \log_2 \left[\frac{S + N}{N + N_R \left(\frac{S}{S + N_R} \right)} \right] \tag{E.2}$$

It may be shown in more direct manner that Eq. (E.2) is a valid expression for the communication rate of the system shown in Fig. 18. For this purpose, the slightly modified system representation shown in Fig. 21 is needed.

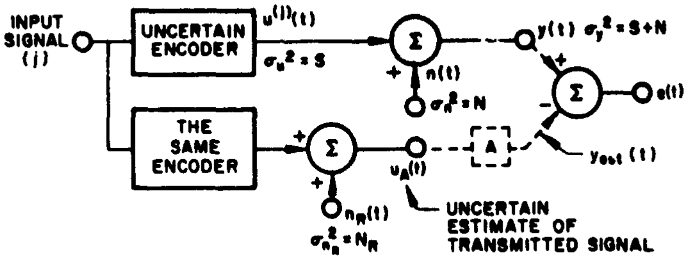


FIG. 21. EQUIVALENT SYSTEM REPRESENTATION EMPLOYING INTEGRAL DIGITAL INPUT.

If j is regarded as a digital input of one out of m possible numbers, the lack of precise reference symbols for receiver signal detection may be precisely represented by an uncertain encoding operation, as indicated. The communication rate is given by

$$R = H(y) - H_j(y) \ , \tag{E.3}$$

where the encoding process is known only to within an uncertain additive signal of variance N_R, as shown in Fig. 21.

If $u_A(t)$ is multiplied by a constant A chosen to minimize the variance of the difference $e(t)$ between $y(t)$ and $y_{est}(t)$, as indicated in Fig. 21, then

$$e(t) = u(t)[1 - A] + n(t) + An_R(t) \tag{E.4}$$

and

$$\sigma_e^2 = S(1 - A)^2 + N + A^2N_R \tag{E.5}$$

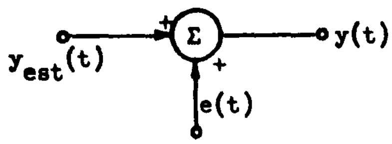
Differentiation of Eq. (E.5) with respect to A and equation of the result to zero yields the optimum value of A. Thus,

$$A = \frac{S}{S + N_R} \tag{E.6}$$

and

$$\sigma_e^2 = N + \frac{SN_R}{S + N_R} \tag{E.7}$$

The relation $y - y_{est} = e$ may be represented by the following diagram:



The product of $y_{est}(t)$ and $e(t)$ may be written by inspection from Fig. 21. Thus,

$$[y_{est}(t)][e(t)] = \Big\{[u(t) + n_R(t)]A\Big\}\Big\{u(t)[1 - A] + n(t) - An_R(t)\Big\} \tag{E.8}$$

Since the different functions of Eq. (E.8) are independent, white, gaussian, random variables, the expected value of the product is

$$\langle[y_{est}(t)]e(t)\rangle = SA(1 - A) - A^2N_R \tag{E.9}$$

Substitution of A from Eq. (E.6) into Eq. (E.9) yields a value of zero, indicating that $e(t)$ is uncorrelated with $y_{est}(t)$. Since both are white, gaussian, random variables, they are necessarily independent, and hence,

$$H_{y_{est}}(y) = W \log[2\pi e \sigma_e^2] = W \log\left[2\pi e\left(N + \frac{SN_R}{S + N_R}\right)\right] \qquad (E.10)$$

Since $y_{est}(t)$ is the best possible estimate of y, given j and the uncertain knowledge of the encoding process, one also has

$$S(y)\big|_j = N + \frac{SN_R}{S + N_R} \qquad (E.11)$$

Because the uncertainty in y, given j, is an independent, white, random, gaussian function, $\bar{S}(y)\big|_j = S(y)\big|_j$, and thus,

$$H_j(y) = W \log\left[2\pi e\left(N + \frac{SN_R}{S + N_R}\right)\right] \qquad (E.12)$$

Substitution of entropy values in Eq. (E.3) yields

$$R = W \log[2\pi e(S + N)] - W \log\left[2\pi e\left(N + \frac{SN_R}{S + N_R}\right)\right]$$

or

$$R = W \log\left[\frac{S + N}{N + N_R\left(\frac{S}{S + N_R}\right)}\right] \qquad (E.13)$$

which agrees precisely with the result of Eq. (E.2). Thus, the validity of the manipulation used to obtain Figs. 19 and 20 is established.

For a channel such as that of Fig. 20, Shannon [Ref. 5] has shown that the ideal detector makes its decision on the basis of the integral-square difference between $y(t)$ and each of the $u'^{(i)}(t)$. In the case of Fig. 18, precisely the same result applies, except that each $u_A^{(i)}$

must be multiplied by the factor

$$\sqrt{\frac{S}{S + N_R}}$$

to obtain the desired reference symbols. Clearly, the resulting integral-square-error-comparison detector is optimum for such a system regardless of whether interpretation is as shown in Fig. 18 or as shown in Fig. 20.

APPENDIX F. ANALYTICAL TECHNIQUES FOR CHANNELS WITH ARBITRARY STOCHASTIC DISTURBANCES

A typical, noisy communication channel with an uncertain time-varying linear transfer function may be regarded as a stochastic signal recoder followed by the addition of independent noise. If the receiver has approximate information concerning the specific channel transfer-function behavior, then it is useful to represent the stochastic recoder by a deterministic (that is, precisely known by the receiver) recoding operation on the transmitted waveform, followed by the addition of an unknown waveform. For example, the certain coding operation might be a known transformation of each possible transmitted-waveform hypothesis $x^{(j)}(t)$ into an estimate $u_{est}^{(j)}(t)$ of the specific received-signal waveform $u^{(j)}(t)$ that would have actually occurred if $x^{(j)}(t)$ had actually been transmitted. The stochastic waveform added by the recorder is, then, simply the estimation error $[u^{(j)}(t) - u_{est}^{(j)}(t)]$.

If a performance analysis of some communication system is undertaken using the channel-model representation discussed in the previous paragraph, correct results obviously must be the same whether the certain recoding operation of $x(t)$ into $u_{est}(t)$ is regarded as part of the channel or simply as part of the transmitter, provided the correct statistical relation between $u(t)$ and $x(t)$ is preserved. By a simple reinterpretation of the system model, a channel with multiplicative disturbances plus additive disturbances can be reduced to a channel with purely additive-noise disturbances. Thus, a large body of well-developed analytical tools for handling additive-noise channels may be borrowed to help in the analysis of channels and systems involving much more general kinds of disturbances.

Use of the above analytical procedure has been found very useful in studying the performance potential of the adaptive matched-filter multipath-communication technique. An investigation of the detection processes involved and of the nature of the reference-symbol estimation procedures has led to the conclusion that the performance of the adaptive matched-filter receiver will be accurately determined by simply assuming that the estimation error is a random, white gaussian noise, independent of the received-symbol estimate, no matter what particular probability distribution it actually may have. This result greatly simplifies the necessary analytical labor, and reduces the amount of statistical channel information required to evaluate system performance. Justification for such an analytical procedure may be effectively presented within the framework of Shannon's geometrical signal-space representation [Ref. 5]. The following arguments are closely related to Shannon's derivation of the channel capacity of a channel with purely independent, white, gaussian, additive-noise disturbances.

Let each of the pertinent system waveforms be represented by a vector in a multidimensional hyperspace, with $2TW$ orthogonal coordinates corresponding to waveform samples taken at time instants separated by $1/2W$. Shannon has shown that a vector corresponding to a W-band-limited waveform of variance N, taken at random from a white gaussian noise, will lie within a distance δ of the surface of a sphere of radius $\sqrt{2TWN}$, with δ arbitrarily small provided T is taken sufficiently large. A similar result may readily be shown (by an argument based on a variation of the central-limit theorem relating to the sums of correlated sets of random variables, where the sets are independent [see Bernstein, Ref. 24]) for waveforms taken at random from much more general kinds of noise sources. The primary restriction on such sources is that intercoordinate influences in output waveforms be limited to a finite maximum time duration. The nature of the assumed signal sources and the finite memory duration of the multipath-channel model considered in Chapter III thus guarantee that the signal vectors $\vec{y}$, $\vec{u}$, $\vec{x}$, $\vec{u}_{est}$, and $\vec{u}_{error}$ will all lie arbitrarily close to very distinct spheres in signal hyperspace, for sufficiently large T.

Let a particular waveform $\vec{x}^{(j)}$ be selected at the transmitter in accordance with message information. In the system interpretation that has been elected, the transmitter recodes $\vec{x}^{(j)}$ into $\vec{u}^{(j)}_{est}$ and transmits the result. The channel first perturbs the transmitted waveform into $\vec{u}^{(j)}$ by adding $[-\vec{u}^{(j)}_{error}]$, and then perturbs the result into $\vec{y}$ by adding $\vec{n}$, an independent, random, white gaussian noise vector of variance N. The distance $|\vec{y} - \vec{u}^{(j)}_{est}|$, which is the rms difference measured by the receiver, is thus equal to $|\vec{n} - \vec{u}^{(j)}_{error}|$. Since $\vec{n}$ is independent of $\vec{u}^{(j)}_{error}$, the measured distance is simply

$$\sqrt{2TW(\sigma^2_{u_{error}} + N \pm \delta)}\ .$$

In making a decision, the receiver also measures each of the distances $|\vec{y} - \vec{u}^{(i)}_{est}|$, for $i \neq j$. If one or more of these distances are less than $|\vec{y} - \vec{u}^{(j)}_{est}|$, a decision error is made.

We note that the operation of the receiver is equivalent to the dividing of signal space into a set of m distinctive spherical-surface regions of uncertainty, each of maximum radius

$$\sqrt{2TW(\sigma^2_{u_{error}} + N + \delta)}\ ,$$

centered at each of the m reference vectors $\vec{u}^{(i)}_{est}$. A received signal $\vec{y}$ will normally be identified as the result of a signal selection $\vec{x}^{(j)}$ if it lies within the sphere of uncertainty centered at reference signal point $\vec{u}^{(j)}_{est}$. Although the visual resemblance to a three-dimensional space is unsatisfactory at this point, it is clear that every signal point $\vec{y}^{(i)}$ that could possibly result from the selection of a particular $\vec{x}^{(i)}$ at the transmitter must lie arbitrarily near the surface of every sphere of radius

$$\sqrt{2TW(\sigma^2_{u_{error}} + N)}$$

centered at every $\vec{u}^{(i)}_{est}$ that could possibly result when waveform $\vec{x}^{(i)}$ is selected at the transmitter. If any received signal $\vec{y}$ lies in a region of signal space common to two spheres, centered at $\vec{u}^{(i)}_{est}$ and $\vec{u}^{(j)}_{est}$, it will be incorrectly identified, for it must lie (in the limit as T increases) closest to the surface of the correct sphere, closest to the center of the incorrect sphere, and thus closest to the incorrect reference vector.

Let us assume first that the waveform $u^{(i)}_{error}(t)$ is taken at random from a white gaussian noise source that is uncorrelated with $\vec{u}^{(i)}_{est}$ and different for each i. Since $n(t)$ is an independent, white gaussian noise, all possible received-signal points on the surface of the sphere of uncertainty (in the limit for large T) centered at the correct $\vec{u}^{(i)}_{est}$ will be equally probable. Therefore, any overlap of any of the spheres of uncertainty will result in the possibility of received signals that will be incorrectly identified. In the case where $\vec{y}$ itself has a uniform spherical probability distribution, it can readily be shown that the number of distinguishable signal vectors can approach arbitrarily close to, but cannot exceed, the ratio of the volume of the $\vec{y}$ sphere to that of the $[\vec{n} - \vec{u}_{error}]$ sphere. The proof is identical with Shannon's derivation [Ref. 5] of channel capacity for an additive independent, white gaussian noise channel, if one simply regards $\vec{u}^{(j)}_{est}$ as the transmitted signal vector. If $\vec{y}$ does not have a uniform spherical distribution, some of the possible regions on its related sphere of radius $\sqrt{2TW\sigma_y^2}$ will be eliminated, with consequent reduction in the volume available for occupancy by the spheres of uncertainty.

Next, consider the type of $\vec{u}^{(i)}_{error}$ that arises in the adaptive matched-filter multipath receiver. The function $\vec{u}^{(i)}_{est}$ is obtained by passing a known waveform $\vec{x}^{(i)}$ through a tapped delay-line filter chosen to minimize the variance of the estimation error, as described in Appendix D. The result corresponds to a constant A multiplied by the waveform $[\vec{u}^{(i)} + \vec{u}^{(i)}_{noise}]$, where $\vec{u}^{(i)}_{noise}$ is a function resulting from the passing of $\vec{x}^{(i)}$ through a delay-line filter with tap multipliers corresponding to errors in measuring the transfer function of the communication channel. Since the tap-multiplier errors are random, zero-mean, and independent of $\vec{x}^{(i)}$, $\vec{u}^{(i)}_{noise}$ is uncorrelated with $\vec{u}^{(i)}$ (though not necessarily independent of $\vec{x}^{(i)}$ nor $\vec{u}^{(i)}$). The correct choice of A, for minimizing the estimation-error variance, is a value $[\sigma_u^2/(\sigma_u^2 + \sigma^2_{u\ noise})]$, and the resulting $\vec{u}^{(i)}_{error}$ is uncorrelated with $\vec{u}^{(i)}_{est}$. These relations may be obtained by procedures very similar to those of Eqs. (E.4) through (E.9).

Since $\vec{u}^{(i)}_{error}$ must still be independent of $\vec{n}$, all possible $\vec{y}^{(i)}$ vectors must still lie arbitrarily close to spherical surfaces of radius $\sqrt{2TW(\sigma^2_{u_{error}} + N)}$ centered at the respective $\vec{u}^{(i)}_{est}$ reference vectors. However, there may now be fissures in these surfaces corresponding to improbable difference vectors. An ideal probability-computing detector could accurately chart the regions of uncertainty such that a received-signal point closest to $\vec{u}^{(j)}_{est}$, but corresponding to an improbable (or in the limit, impossible) difference vector $[\vec{n} - \vec{u}^{(j)}_{error}]$, could be correctly identified with some other reference symbol which was farther away. Thus, the regions of uncertainty could be packed closer together without error in identifying $\vec{y}$. This is just what can happen whenever a simple additive-noise communication channel has gaussian-noise statistics that are nonwhite, but the ideal detection process can be simplified by performing a transformation on signal hyperspace which leaves the noise vectors with a uniform, spherical, probability distribution (i.e., by processing signals with a "noise-whitening" filter). A simple distance-measuring detector then becomes optimum.

Note, however, that if a distance-measuring detector is used without the noise-whitening transformation, there can be no assurance that every possible $\vec{y}$ can be correctly identified unless there is no overlap in spheres tangent to the actual regions of uncertainty. It is conceivable that a special situation could exist wherein the true regions of uncertainty were all external to the common volumes of overlapping tangential spheres, so that the integral-squared-error comparison detector could be optimum without the need for prewhitening. The possibility of such a situation occurring in the case of a multipath-channel receiver appears to be slim. To guarantee correct identification of $\vec{y}$ with the proposed form of detector, one must apparently deny the possibility of overlap of the spheres surrounding the regions of uncertainty. Such a restriction imposes the same constraints on system performance as would result if $u^{(i)}_{error}(t)$ were indeed an independent, white, gaussian, random variable. Note also, that this restriction guarantees that, in the limit as T increases, the system performance will be as good as it would be if $u_{error}(t)$ were white, gaussian random noise, no matter what particular probability distribution may actually occur.

If one were to try to build a better detector, in connection with an adaptive matched-filter multipath receiver, a likely procedure would be to construct whitening filters for the differences $[\vec{n} - u^{(i)}_{error}]$, and to process appropriate pairs of waveforms with these filters prior to the integral-squared-error detection process. Such a procedure would be readily possible if the estimation error were gaussian, but in more general situations there can be no transformation that will result in the desired random, white, gaussian probability distribution. If it is assumed that the estimation error be a gaussian random process (or very nearly so), a consideration of the energy spectrum of $\vec{x}^{(i)}$ and of the nature of the random linear transformation that generates u_{error} shows that it would have to be specifically white and gaussian. It is presently difficult to imagine any sort of transformation on $[n - \vec{u}^{(i)}_{error}]$ which would benefit system performance in a practical situation.

While the foregoing considerations go a long way toward justifying calculations based on the substitution of white, gaussian random noise for $\vec{u}^{(i)}_{error}(t)$, there is a powerful independent justification for this step. Consider the vector $[\vec{n} - \vec{u}^{(j)}_{error}]$, where $\vec{n}$ has an independent, white gaussian probability distribution and is large compared to $\vec{u}_{error}$. It has been shown by Shannon [Ref. 4] that the probability distribution of such a vector sum is, to a very good approximation, the same as that of a purely white gaussian random vector, because of the innate ability of white gaussian noise to "absorb" certain kinds of small random variables that do not have white gaussian statistics. For the conditions described, the region of uncertainty about each $\vec{u}^{(i)}_{est}$ is affected very little by the particular statistics that $\vec{u}_{error}$ may exhibit, and performance calculations based on the assumption of a uniform spherical distribution should yield results very close to those based on exact knowledge of the specific probability distribution of $\vec{u}_{error}$.

An investigation of the various system operating conditions discussed and plotted in Chapter III reveals that $\sigma^2_{u_{error}} \ll N$ in all cases except where the channel parameter product T_mB_s becomes very large. As T_mB_s becomes large, however, the probability distribution of $\vec{u}_{error}$ itself must surely approach that of an independent, white, gaussian random variable. Therefore, the substitution that has been made for analytical purposes should give good results under all the conditions that have been considered.

REFERENCES

1. R. Price, "Statistical Theory Applied to Communication Through Multipath Disturbances," TR No. 266, Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Mass., 3 Sep 1953.
2. G. L. Turin, "Communication Through Noisy, Random-Multipath Channels," TR No. 116, Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, Mass., 14 May 1956.
3. T. Kailath, "Correlation Detection of Signals Perturbed by a Random Channel," IRE Trans. Information Theory, IT-6, Jun 1960, pp. 361-366.
4. C. E. Shannon and W. Weaver, The Mathematical Theory of Communication, The University of Illinois Press, Urbana, Ill., 1949.
5. C. E. Shannon, "Communication in the Presence of Noise," Proc. IRE, vol. 37, Jan 1949, pp. 10-21.
6. R. Price, "Error Probabilities for the Ideal Detection of Signals Perturbed by Scatter and Noise," Group Rep. No. 34-40, Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, Mass., 3 Oct 1955.
7. T. Kailath, "Sampling Models for Linear Time-Variant Filters," TR No. 352, Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Mass., May 1959.
8. R. Price and P. E. Green, Jr., "A Communication Technique for Multipath Channels," Proc. IRE, 46, Mar 1958, pp. 555-570.
9. S. O. Rice, "Communication in the Presence of Noise," Bell Sys. Tech. J., 29, Jan 1950, pp. 60-93.
10. D. P. Harris, "Multipath and Scatter Communication Techniques," IRE 1962 Int. Conv. Rec., 10, Part VIII, Mar 1962.
11. V. C. Pineo, L. G. Kraft, and H. W. Briscoe, "Ionospheric Backscatter Observations at 400 Mc./S," J. Geophys. Res., 65, May 1960, pp. 1620-1621.
12. Tor Hagfors, "Density Fluctuations in a Plasma in a Magnetic Field, with Applications to the Ionosphere," J. Geophys. Res., 66, Jun 1961, pp. 1699-1712.
13. A. M. Peterson, "Free Electron Scatter as a Communication Mode," Paper presented at IRE 1961 Western Electronic Show and Convention, San Francisco, Calif., 22-25 Aug 1961.
14. W. E. Morrow, Jr., "Orbital Scatter Communication," Paper presented at the XIIIth General Assembly of the International Scientific Radio Union, London, England, 5-15 Sep 1960.
15. J. N. Pierce, "Theoretical Limitations on Frequency and Time Diversity for Fading Binary Transmissions," IRE Trans. Communications Systems, CS-9, 2, Jun 1961, p. 186.

REFERENCES (Cont'd)

16. D. P. Harris, "Techniques for Incoherent Scatter Communication," Paper presented at IRE 1961 Western Electronic Show and Convention, San Francisco, Calif., 22-25 Aug 1961.

17. H. J. Juda and F. R. Skalbania, "Frequency-Error Effects on a Binary Communication System Utilizing Coherent Integrators," IRE Record of the Seventh National Communication Symposium, Utica, N. Y., 2-4 Oct 1961.

18. B. M. Oliver, J. R. Pierce, and C. E. Shannon, "The Philosophy of PCM," Proc. IRE, 36, Nov 1948, pp. 1324-1331.

19. J. G. Lawton, "Comparison of Binary Data Transmission Systems," IRE-PGME Proc. of the Second National Convention on Military Electronics, Jun 1958, pp. 54-61.

20. M. L. Doelz, E. T. Heald, and D. L. Martin, "Binary Data Transmission Techniques for Linear Systems," Proc. IRE, 45, May 1957, pp. 656-661.

21. N. M. Abramson, "A Class of Systematic Codes for Non-Independent Errors," IRE Trans. Information Theory, IT-5, 4, Dec 1959, pp. 150-156.

22. S. H. Reiger, "Codes for the Correction of Clustered Errors," IRE Trans. Information Theory, IT-6, 1, Mar 1960, pp. 16-20.

23. D. Bierens De Haan, Nouvelles Tables D'Integrales Definies, Hafner Publishing Company, New York, N. Y., 1957.

24. S. Bernstein, "Sur l'extension du theoreme limite du calcul des probabilites aux sommes des quantites dependantes," Math. Annals, 97, 1926, p. 1.